

Electromagnetic proton form factors for neutrino physics and atomic spectroscopy



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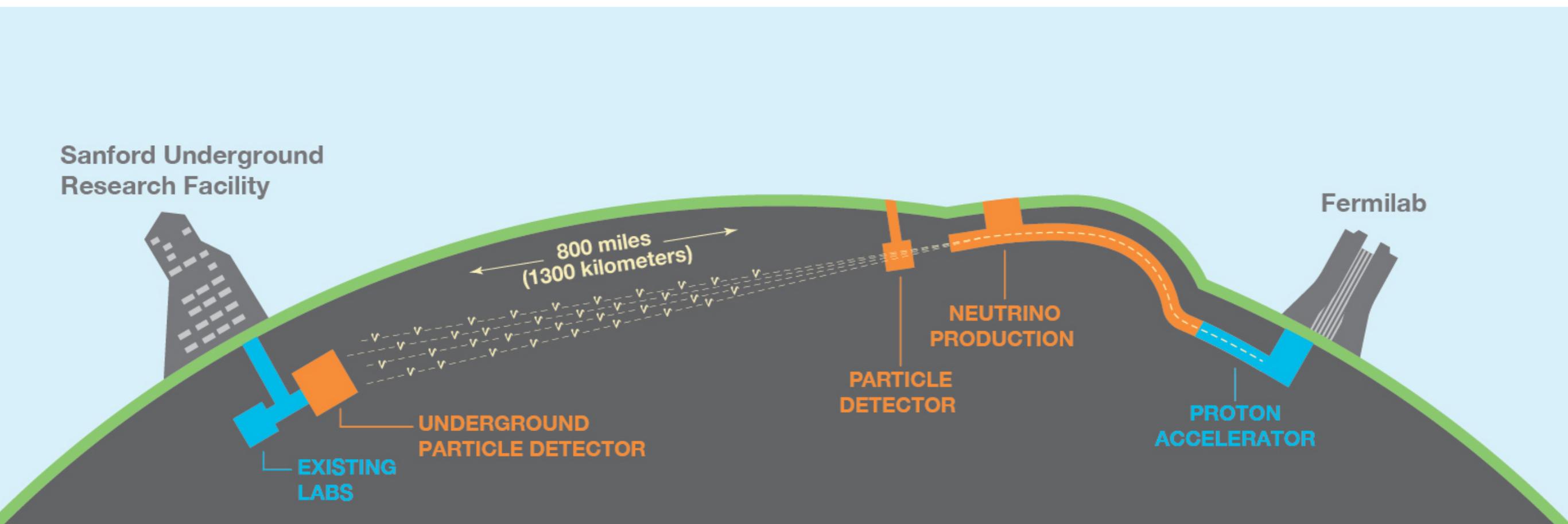
Kaushik Borah, Richard J. Hill, Gabriel Lee and O. T. (arXiv: 2003.13640)

Outline

- 1) electromagnetic form factors at low energies:
lepton-nucleon scattering and atomic spectroscopy
- 2) electron-proton scattering:
z-expansion fits and improved radiative corrections
- 3) neutrino-nucleon CCQE cross sections:
inclusion of neutron and high-precise A1@MAMI data
- 4) nucleon structure corrections to energy levels:
 $1S$ HFS in muonic and $1S-2S$ in ordinary hydrogen

Neutrino experiments

- DUNE and Hyper-K: leading-edge ν science experiments



- measurement of ν_μ disappearance and ν_e appearance from count rates

$$N_\nu \sim \int d\omega \Phi_\nu(\omega) \times \sigma(\omega) \times R(\omega, \omega^{\text{rec}})$$

- cross sections enter neutrino oscillation analysis

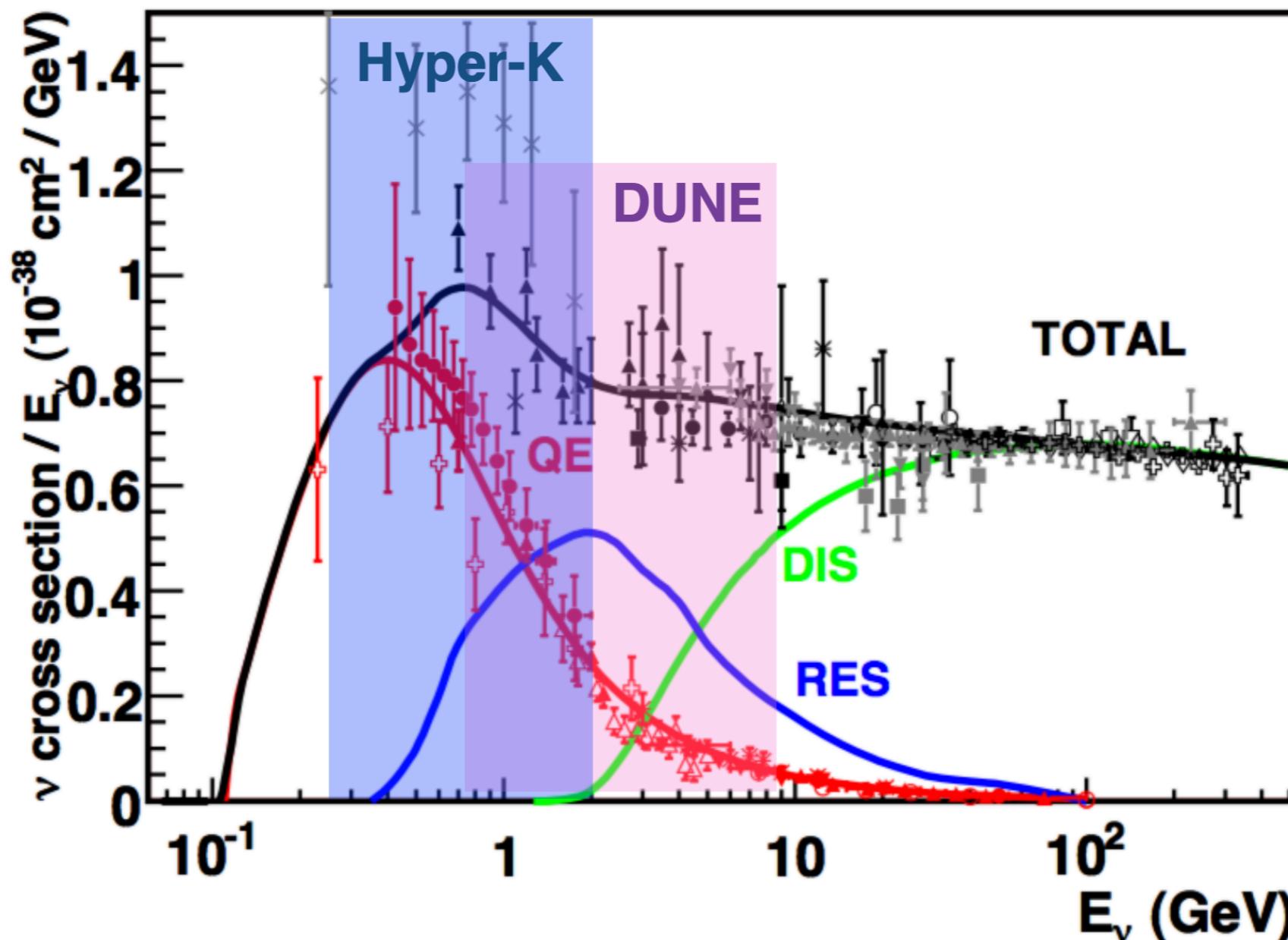


(Anti)neutrino-nucleon charged-current quasielastic scattering



CCQE. Why should we care?

- neutrino-nucleus cross sections and future accelerator-based fluxes

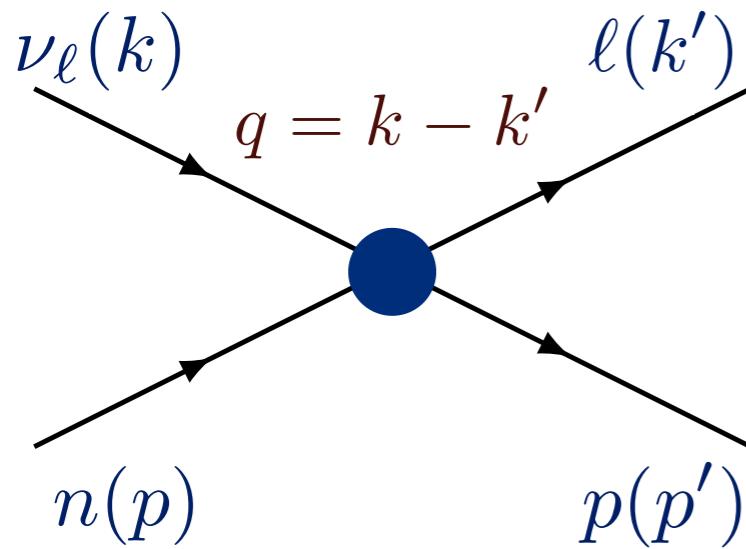


Noemi Rocco

talk at Neutrino 2020

- basic process: bulk of events at Hyper-K and DUNE
- best channel for reconstruction of neutrino energy

CCQE scattering on free nucleon



neutrino energy

$$E_\nu$$

momentum transfer

$$Q^2 = -q^2$$

contact interaction at GeV energies

- assuming isospin symmetry, nucleon current:

$$\Gamma^\mu(Q^2) = \gamma^\mu F_D^V(Q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2M} F_P^V(Q^2) + \gamma^\mu \gamma_5 F_A(Q^2) + \frac{q^\mu}{M} \gamma_5 F_P(Q^2)$$

form factors: isovector Dirac and Pauli

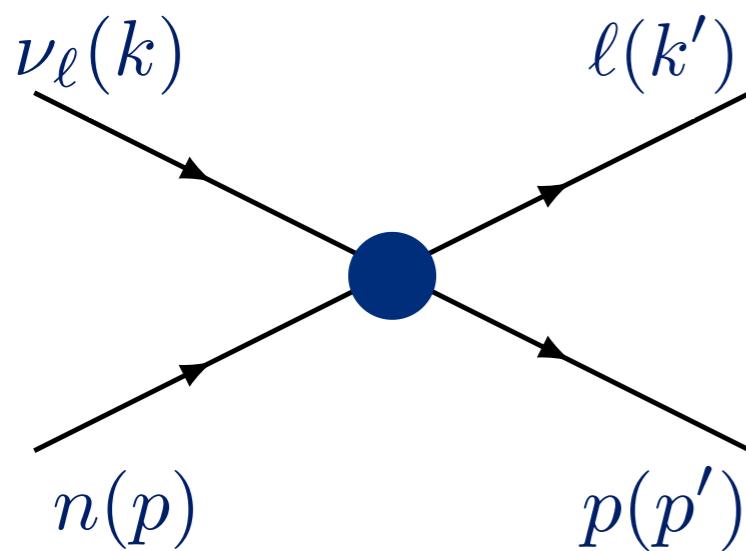
axial and pseudoscalar

$$F_{D,P}^V = F_{D,P}^p - F_{D,P}^n$$

tree-level amplitude

$$T = \frac{G_F V_{ud}}{\sqrt{2}} (\bar{\ell}(k') \gamma_\mu (1 - \gamma_5) \nu_\ell(k)) (\bar{p}(p') \Gamma^\mu(Q^2) n(p))$$

CCQE scattering on free nucleon



$$s - u = 4ME_\nu - Q^2 - m_\ell^2$$
$$\tau = \frac{Q^2}{4M^2}$$

unpolarised observables are measured
cross section expression:

$$\frac{d\sigma}{dQ^2} \sim \frac{M^2}{E_\nu^2} \left(A(Q^2) \frac{m_\ell^2 + Q^2}{M^2} - B(Q^2) \frac{s - u}{M^2} + C(Q^2) \left(\frac{s - u}{M^2} \right)^2 \right)$$

Llewellyn Smith

- structure-dependent functions:

$$A = 2\tau(F_D^V + F_P^V)^2 - (1 + \tau) \left[(F_D^V)^2 + \tau(F_P^V)^2 - (F_A)^2 \right]$$
$$- \frac{m_\ell^2}{4M^2} \left[(F_D^V + F_P^V)^2 + (F_A + 2F_P)^2 - 4(1 + \tau)F_P^2 \right]$$

$$B = 4\tau F_A (F_D^V + F_P^V) \quad C = \frac{1}{4} \left[(F_D^V)^2 + \tau(F_P^V)^2 + (F_A)^2 \right]$$

- cross section is sensitive to both vector and axial contributions

CCQE scattering on free nucleon

- only 3 experiments performed with deuterium bubble chamber
direct access to form-factor shape

ANL 1982: 1737 events

BNL 1981: 1138 events

FNAL 1983: 362 events

world data: ~3200 events

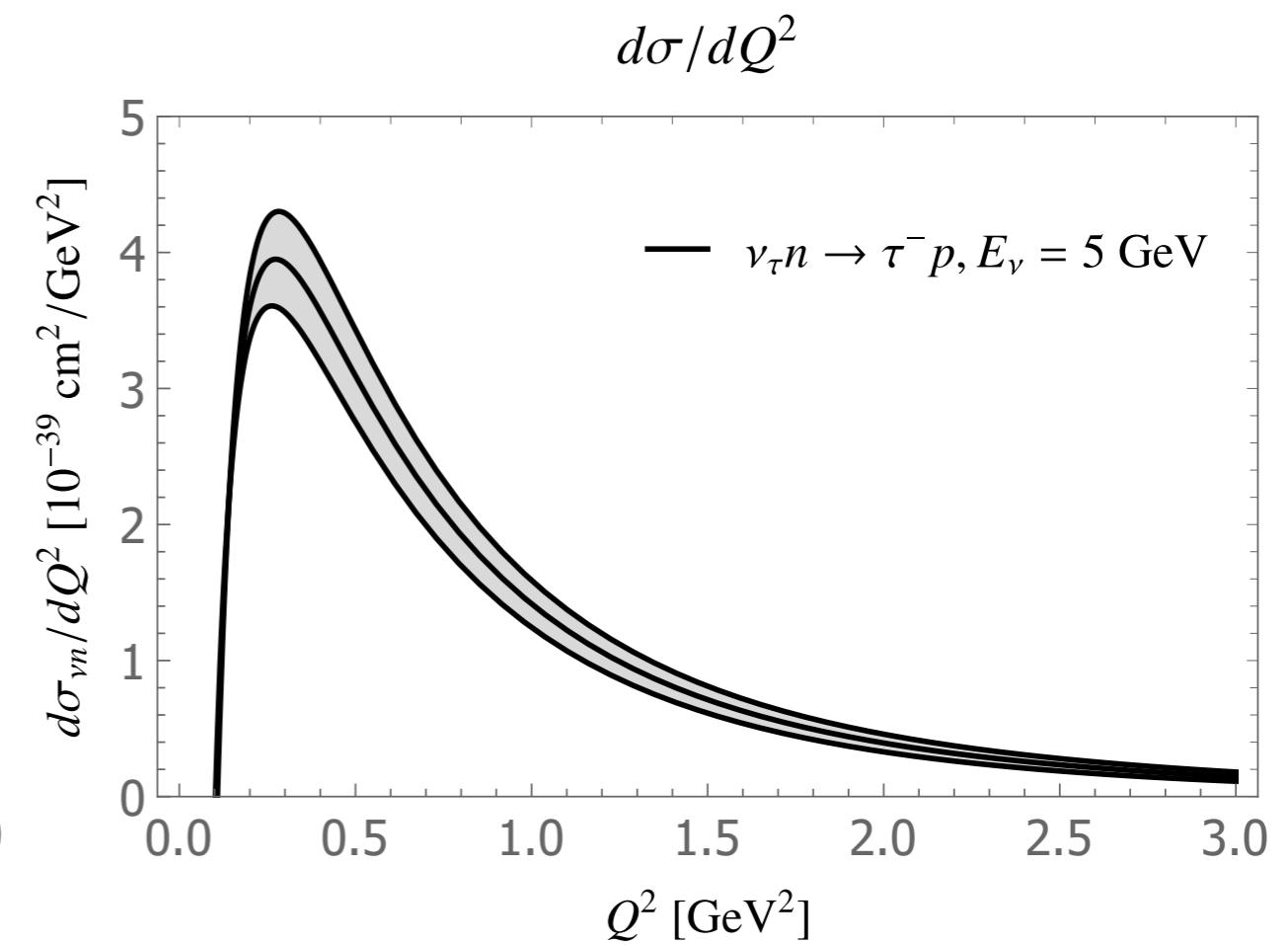
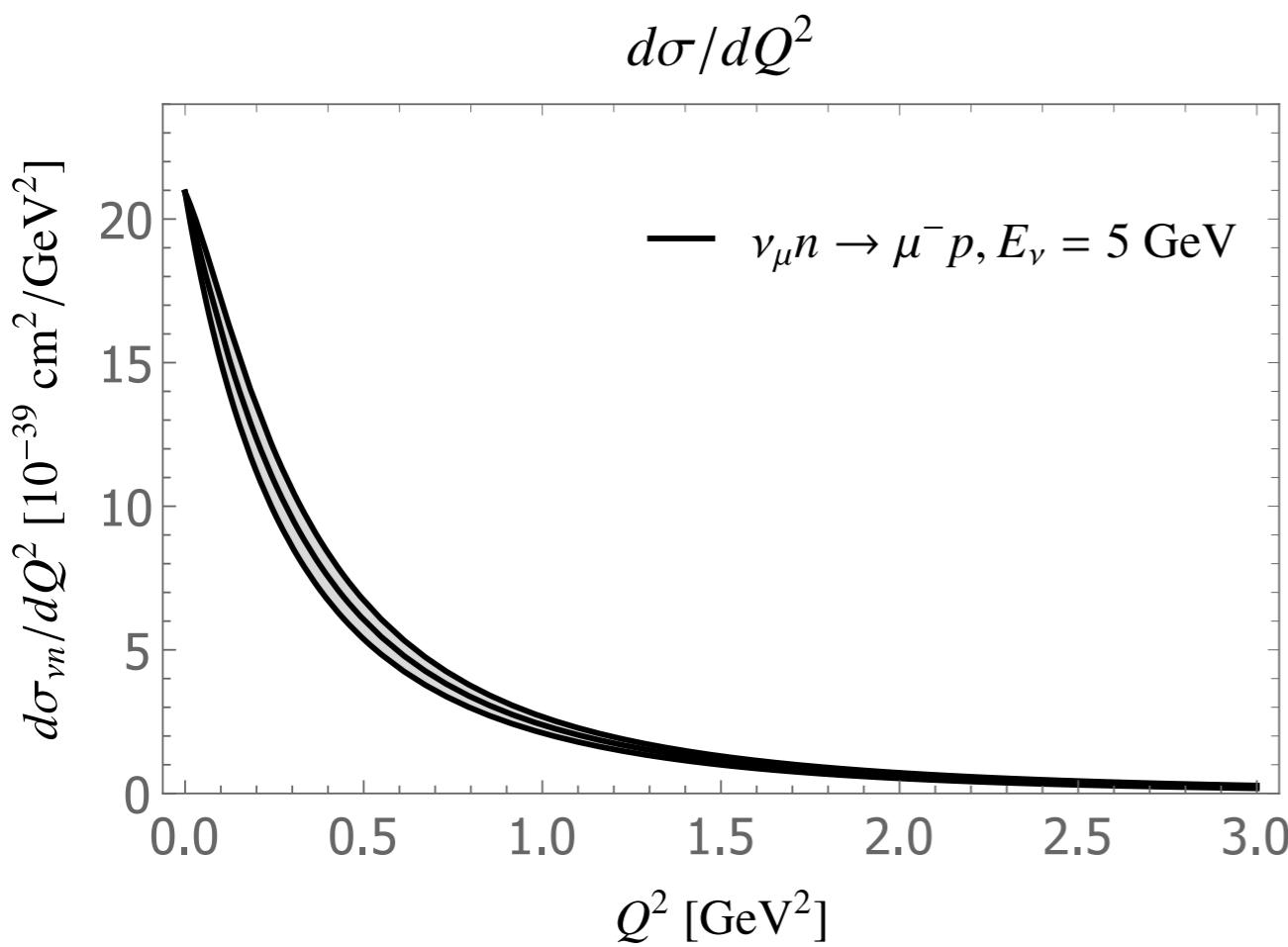


Fermilab bubble chamber, Richard Drew

- axial form factor extracted based on electromagnetic structure

CCQE scattering cross section

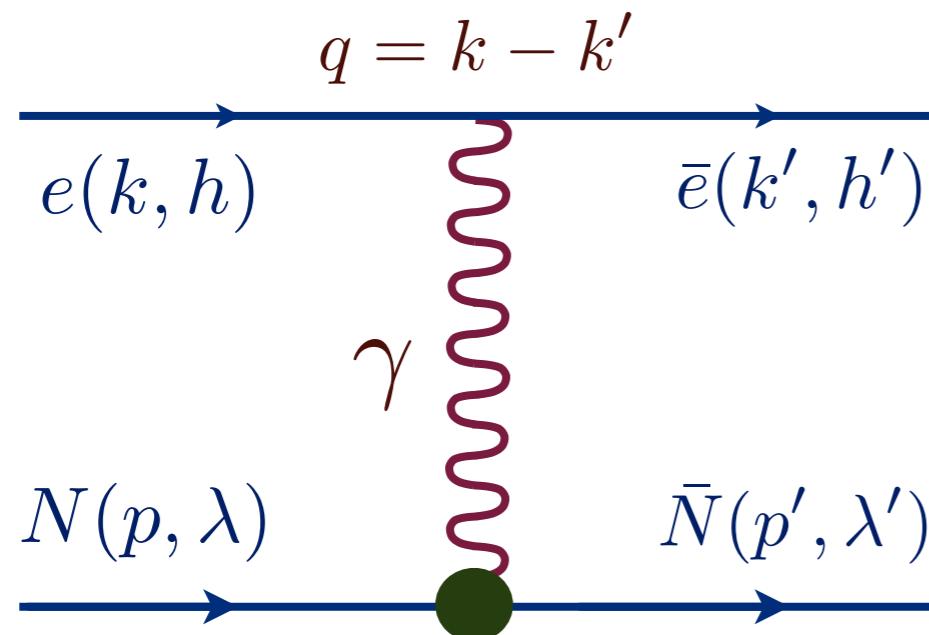
- cross section is energy-independent above a few GeV neutrino energy
- relevant kinematic range of structure effects:



- cross section is dominated by low Q^2 region

Electromagnetic form factors

Tool to explore the proton structure



photon-proton vertex

$$\Gamma^\mu(Q^2) = \gamma^\mu F_D(Q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2M} F_P(Q^2)$$

Dirac and Pauli form factors

lepton energy

E

momentum transfer

$$Q^2 = -(k - k')^2$$

1γ amplitude

$$T = \frac{e^2}{Q^2} (\bar{e}(k', h') \gamma_\mu e(k, h)) \cdot (\bar{N}(p', \lambda') \Gamma^\mu(Q^2) N(p, \lambda))$$

Form factors measurement

Sachs electric and magnetic form factors

$$G_E = F_D - \tau F_P \quad G_M = F_D + F_P$$

Rosenbluth separation

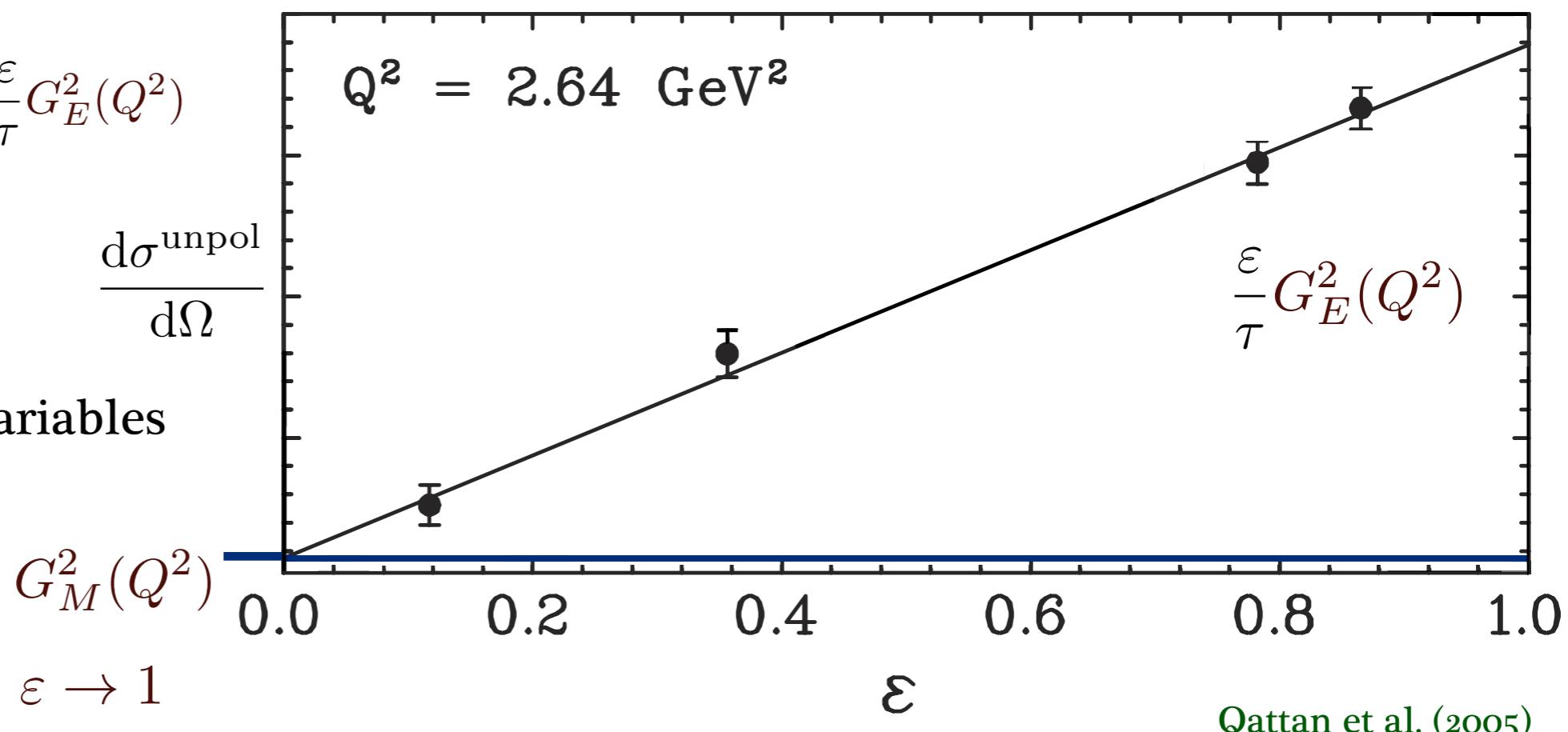
$$\frac{d\sigma^{\text{unpol}}}{d\Omega} \sim G_M^2(Q^2) + \frac{\varepsilon}{\tau} G_E^2(Q^2)$$

$$\tau = \frac{Q^2}{4M^2}$$

τ, ε kinematical variables

$$\varepsilon \leftrightarrow \theta_{\text{lab}}$$

forward scattering: $\varepsilon \rightarrow 1$



Qattan et al. (2005)

- Rosenbluth slope is sensitive to corrections beyond 1γ

Form factors measurement

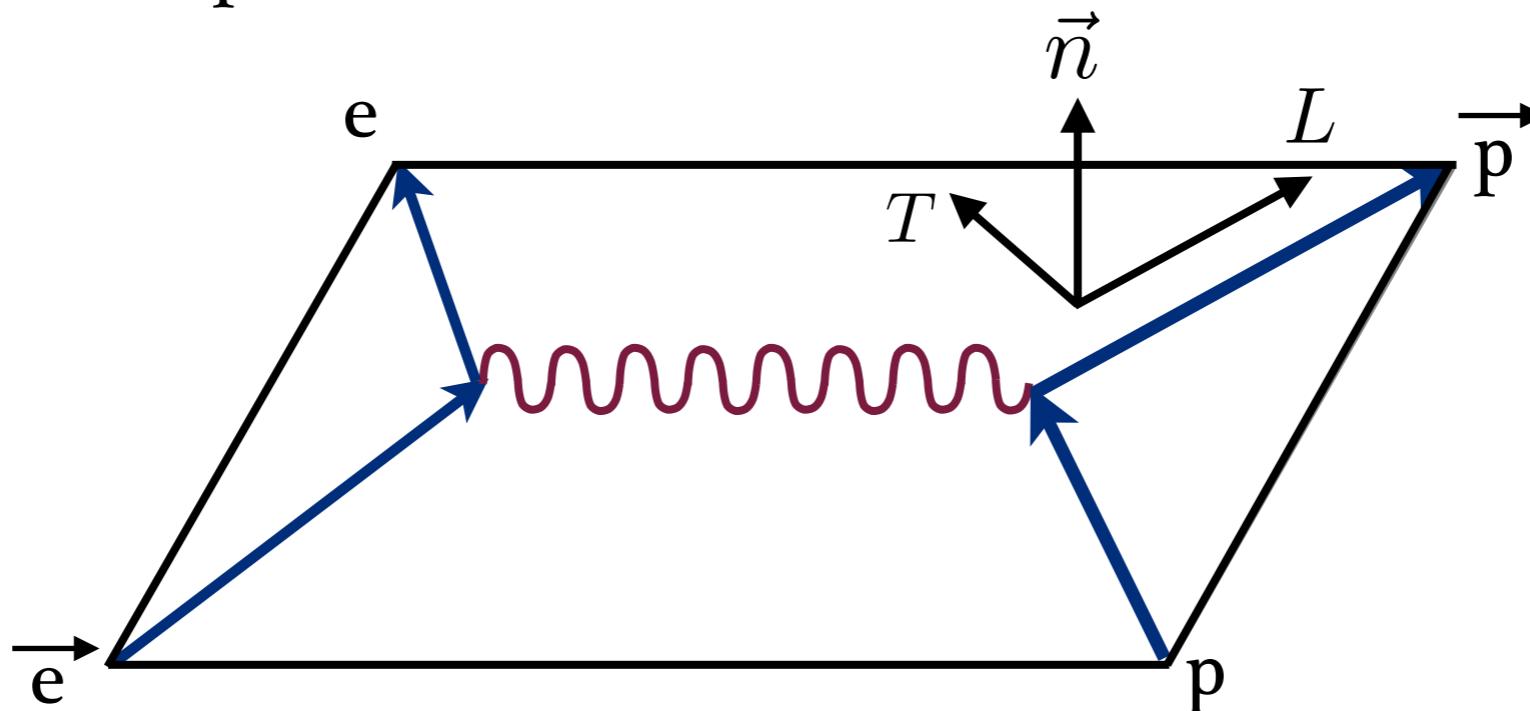
Sachs electric and magnetic form factors

$$G_E = F_D - \tau F_P \quad G_M = F_D + F_P$$

polarization transfer

$$\vec{e} + p \rightarrow e + \vec{p}$$

realized in 2000



$$P_T \sim G_E(Q^2)G_M(Q^2)$$

$$P_L \sim G_M^2(Q^2)$$



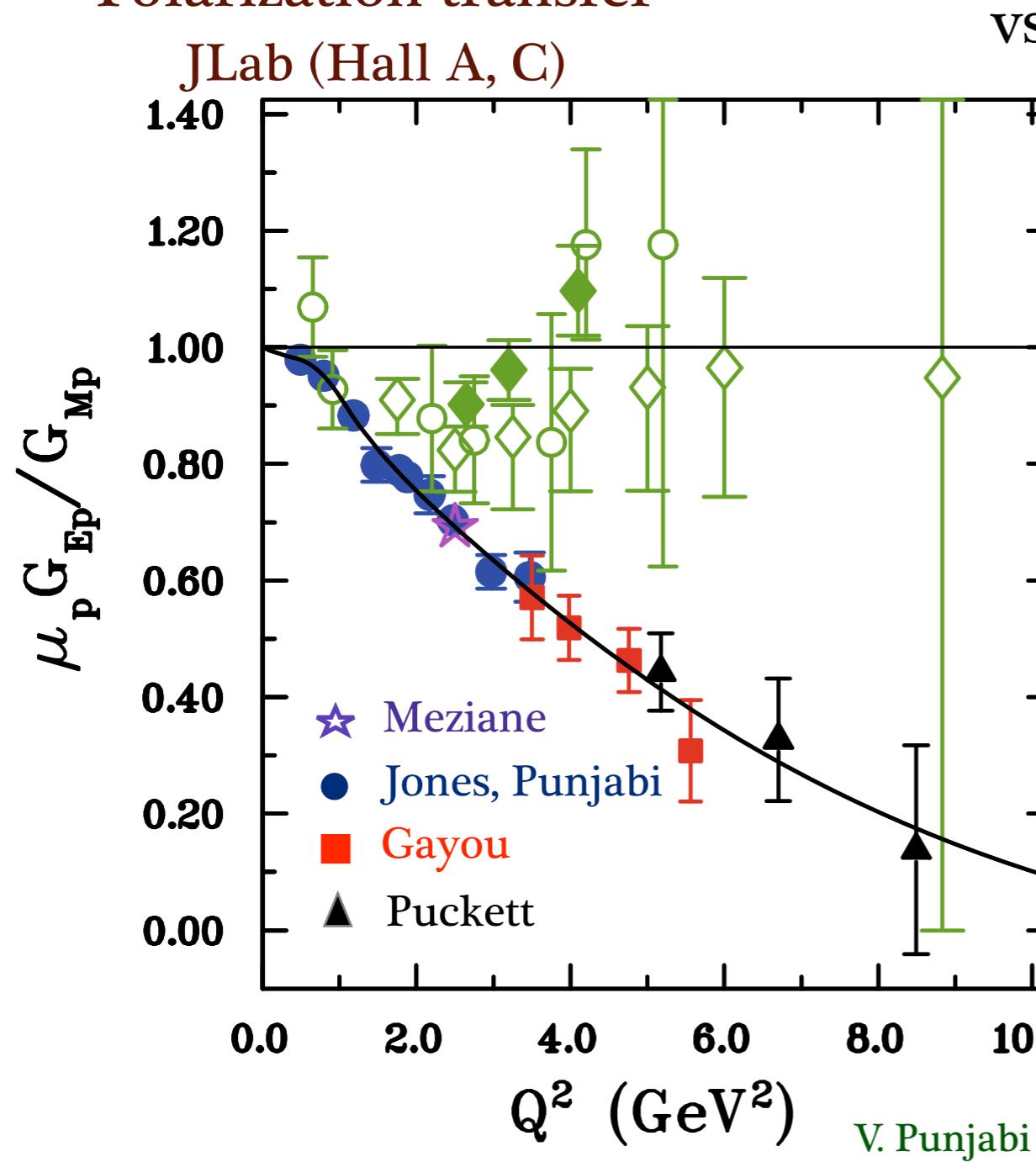
$$\frac{P_T}{P_L} \sim \frac{G_E(Q^2)}{G_M(Q^2)}$$

Proton form factors puzzle

Polarization transfer

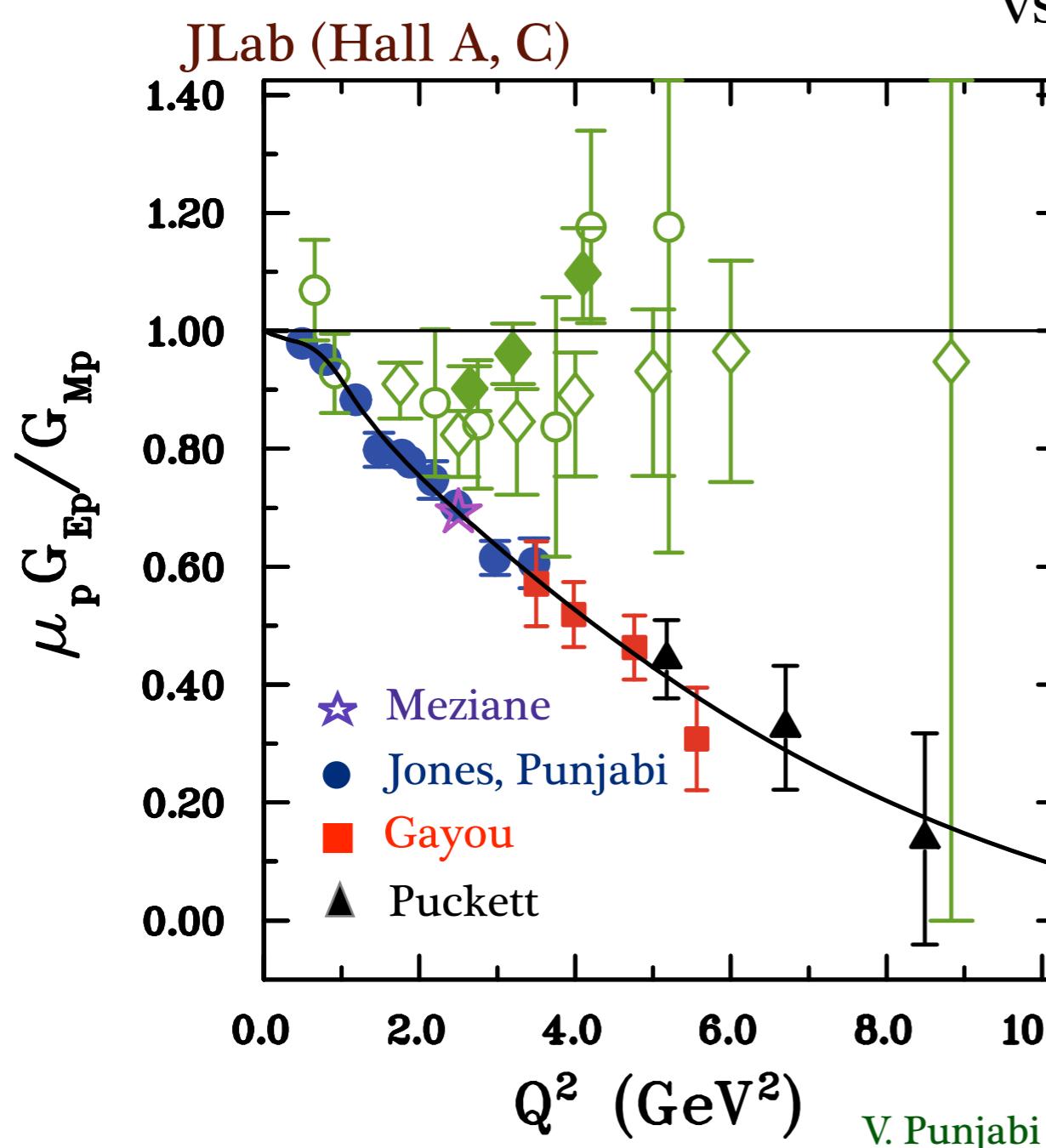
Rosenbluth separation

SLAC, JLab (Hall A, C)



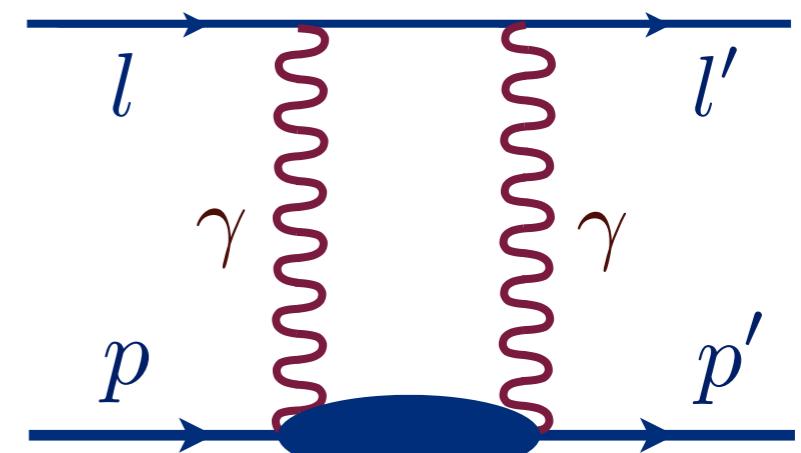
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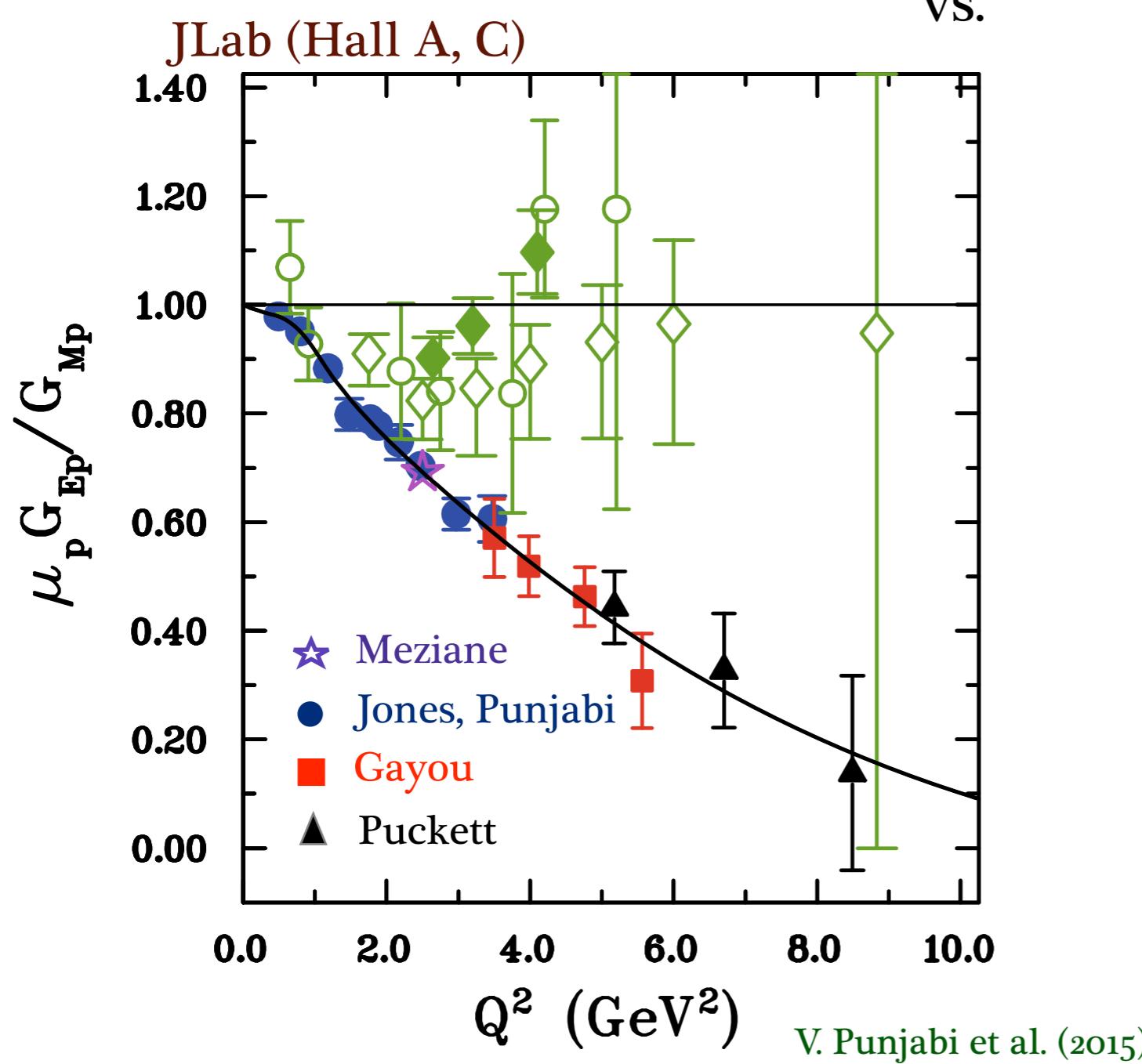
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possible explanation
two-photon exchange

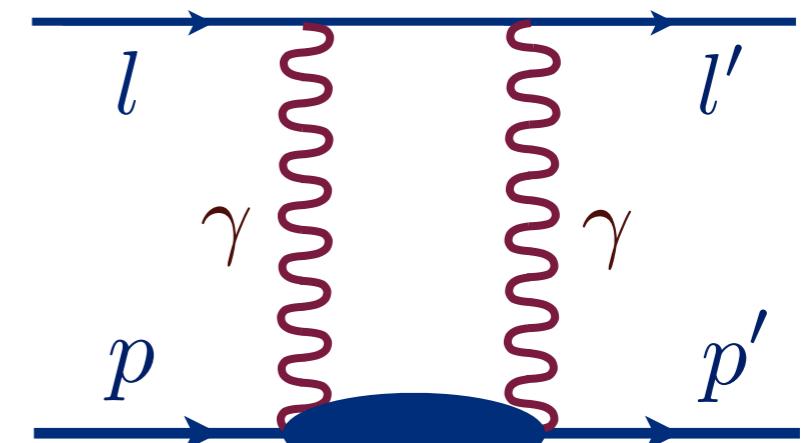
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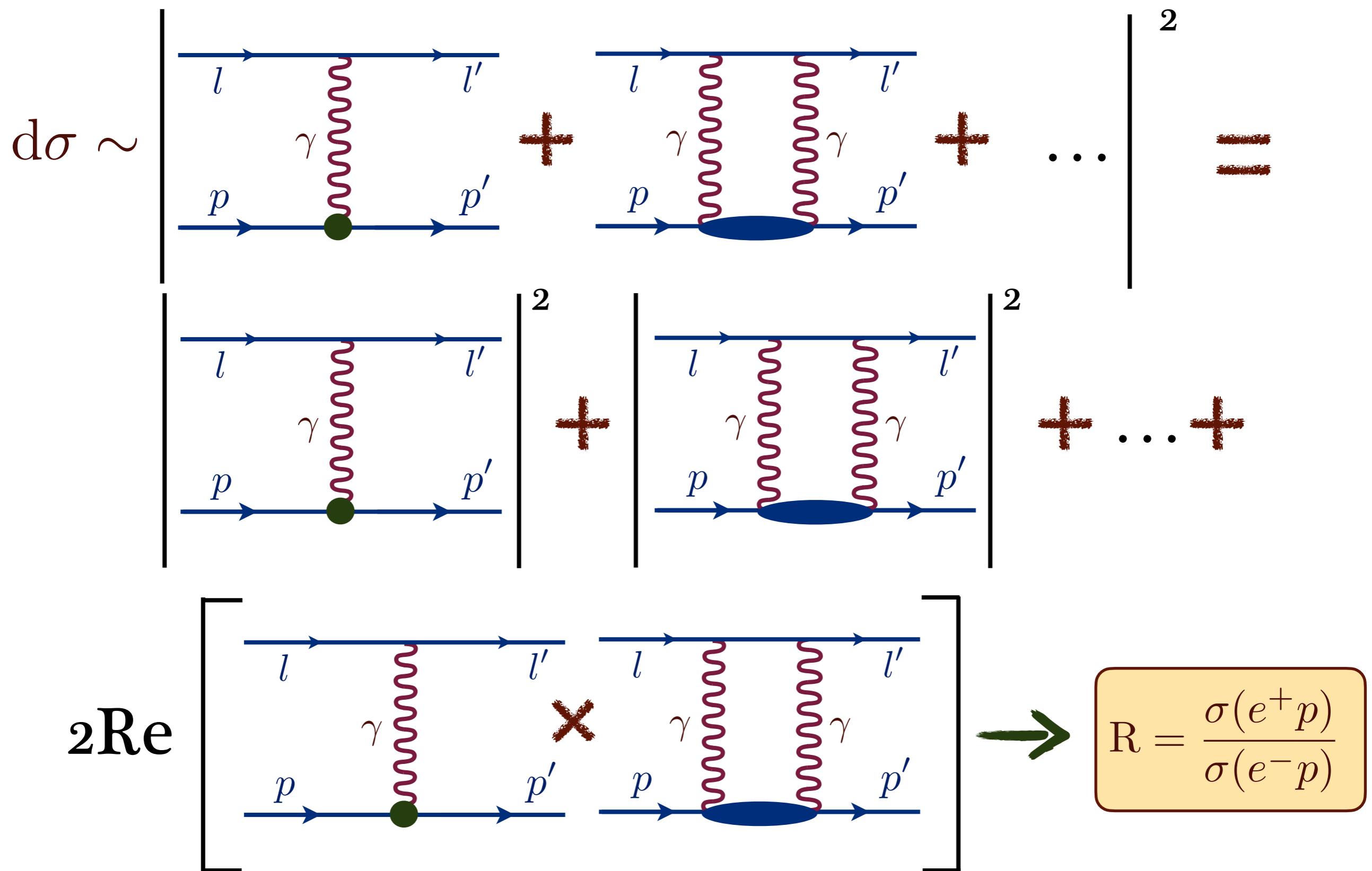


new 2γ measurements

VEPP-3, CLAS@JLAB, OLYMPUS@DESY

- discrepancy motivates study of 2γ

2χ from experiment



Proton form factors puzzle

- problem is not completely solved !!!
- interest in high- Q^2 measurements of R

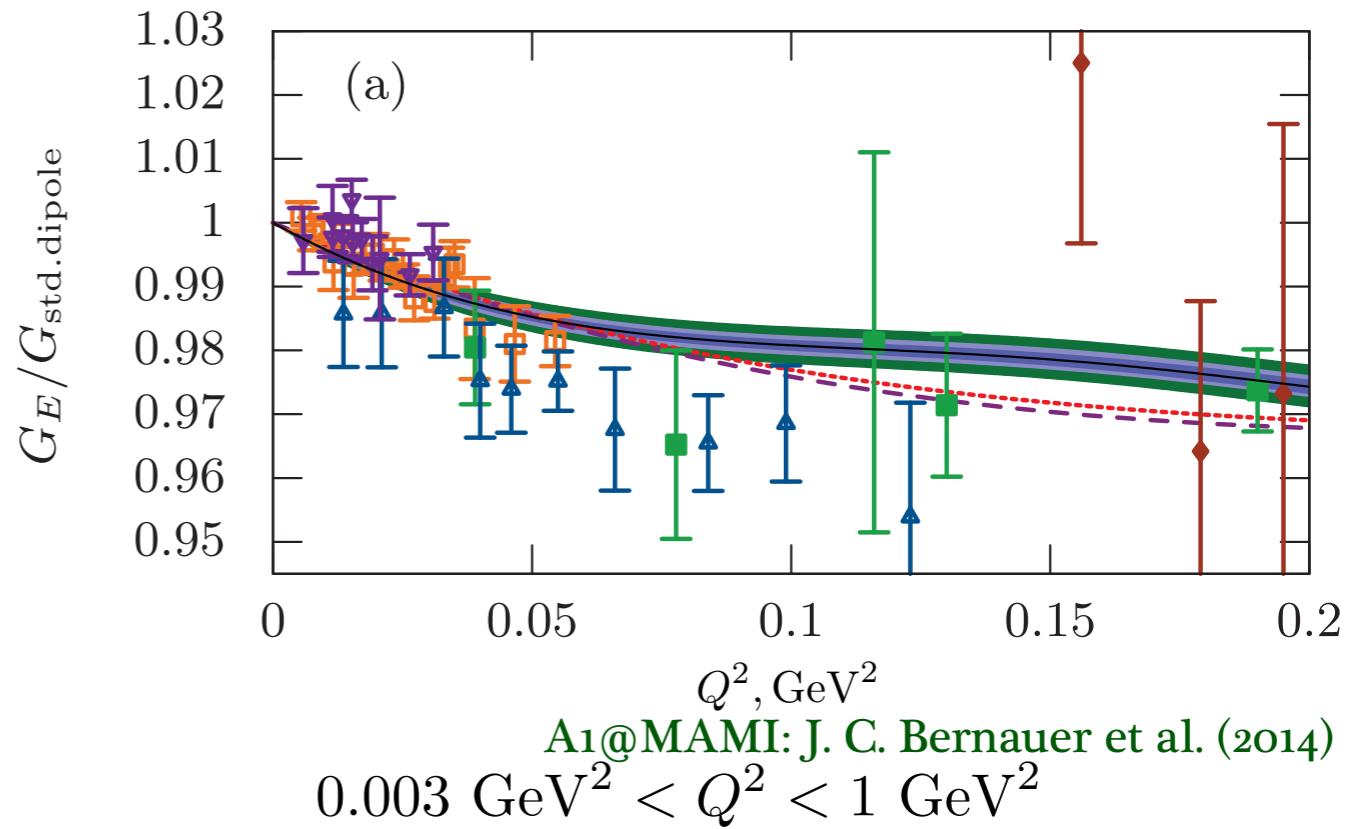
Proton radius

electric charge radius

$$\langle r_E^2 \rangle \equiv -6 \left. \frac{dG_E(Q^2)}{dQ^2} \right|_{Q^2=0}$$

- ep elastic scattering

$$r_E = 0.879 \pm 0.008 \text{ fm}$$



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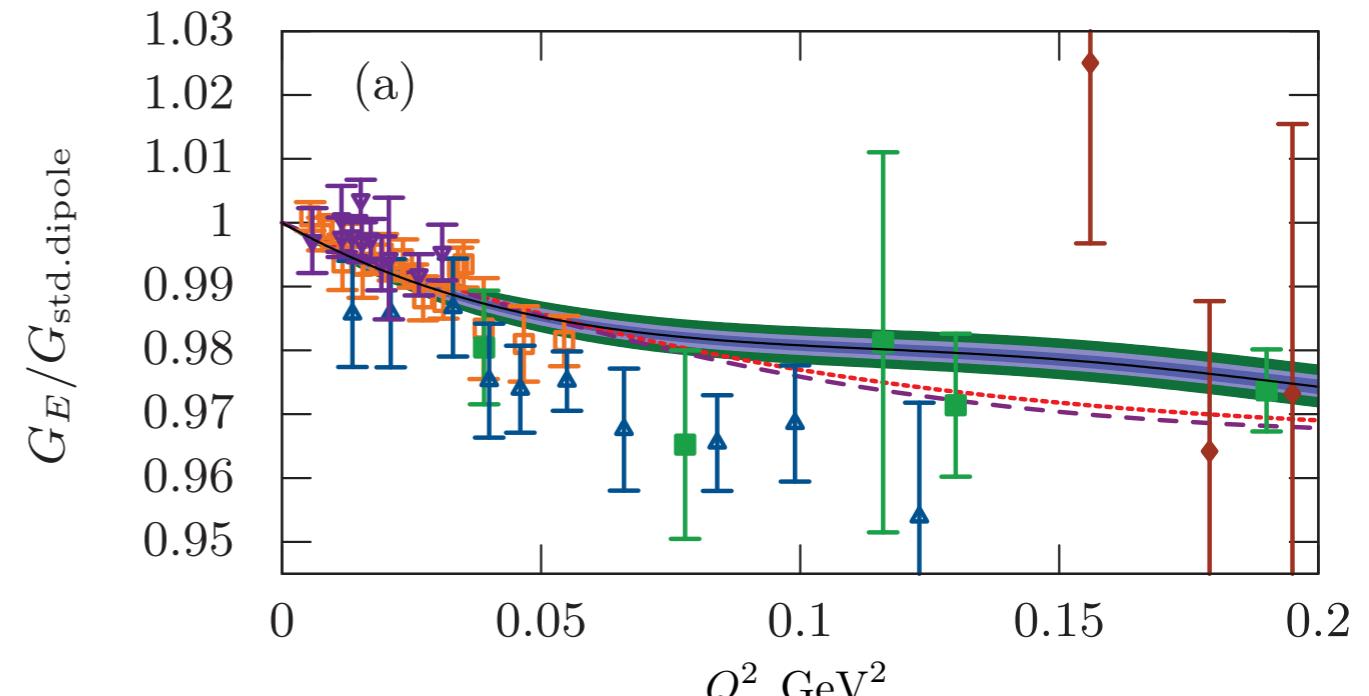
- atomic spectroscopy

$$\nu_{nS} = -\frac{R_\infty}{n^2} + \frac{c_r m_r^3 \langle r_E^2 \rangle}{n^3} + \dots$$

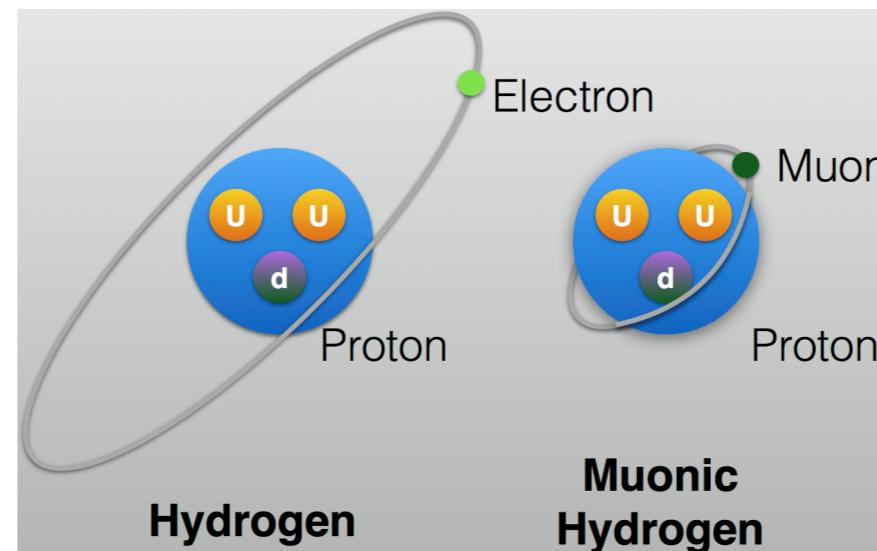
H, D spectroscopy

$$r_E = 0.8758 \pm 0.0077 \text{ fm}$$

CODATA 2010



A1@MAMI: J. C. Bernauer et al. (2014)



μ H Lamb shift

$$r_E = 0.8409 \pm 0.0004 \text{ fm}$$

CREMA (2010, 2013)

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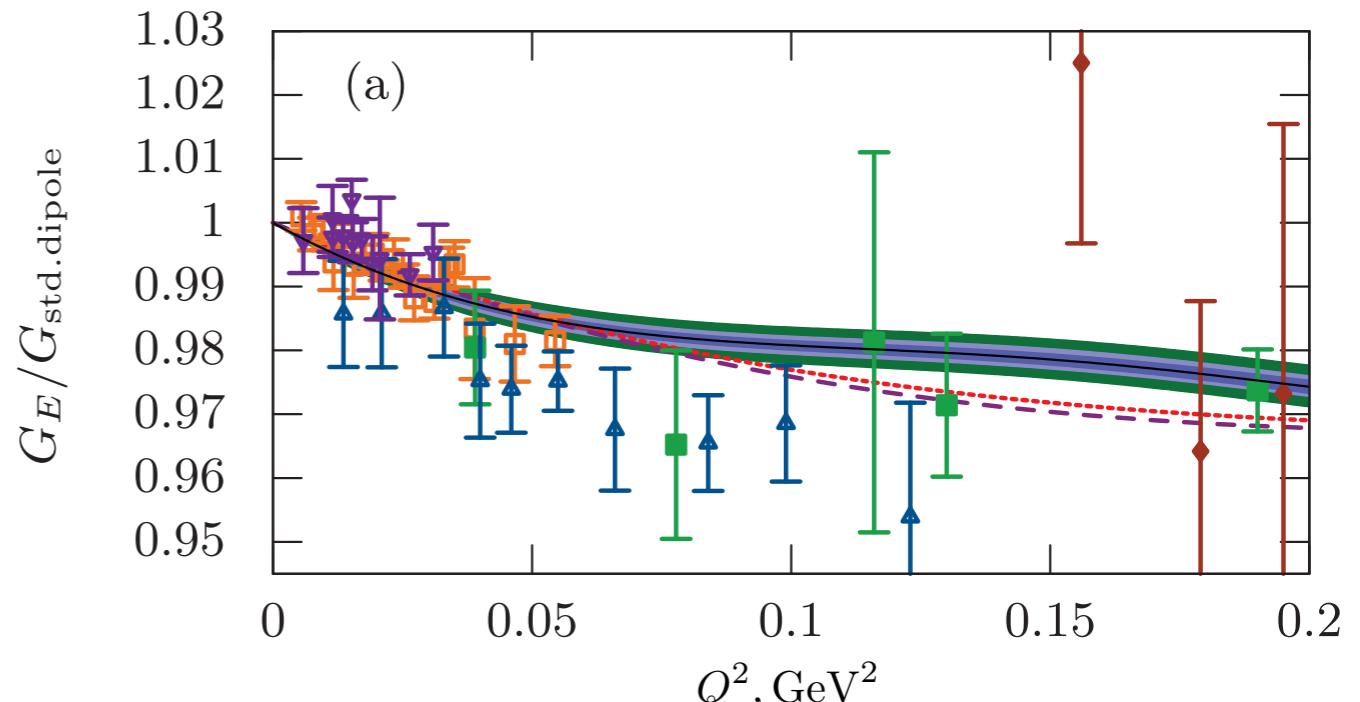
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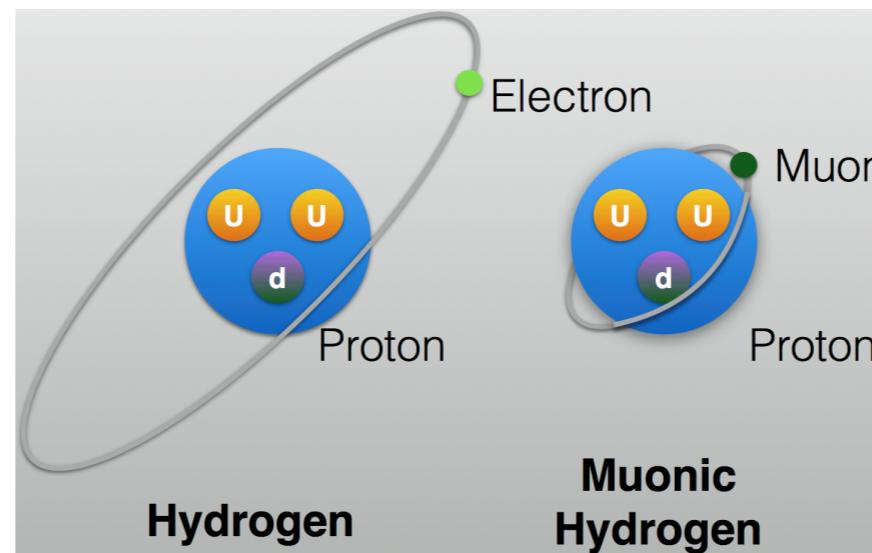
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5.6 σ difference !



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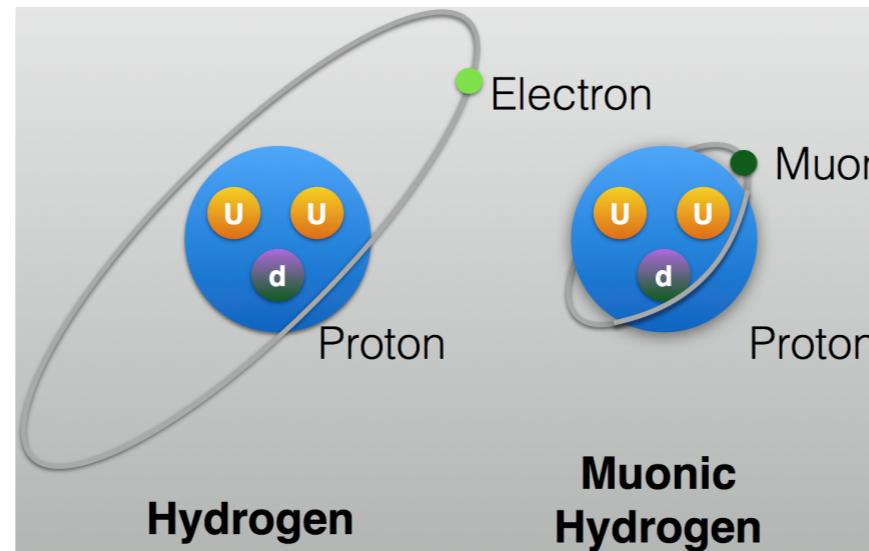
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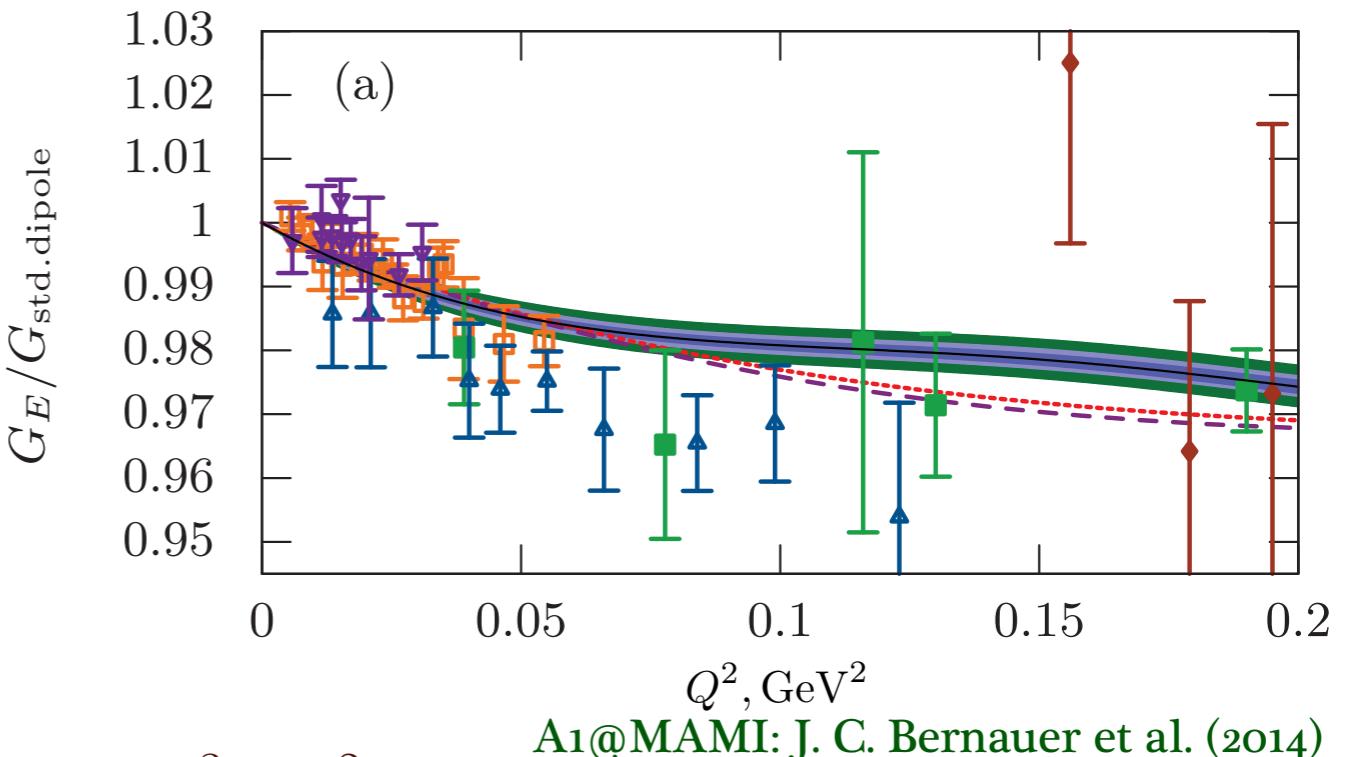
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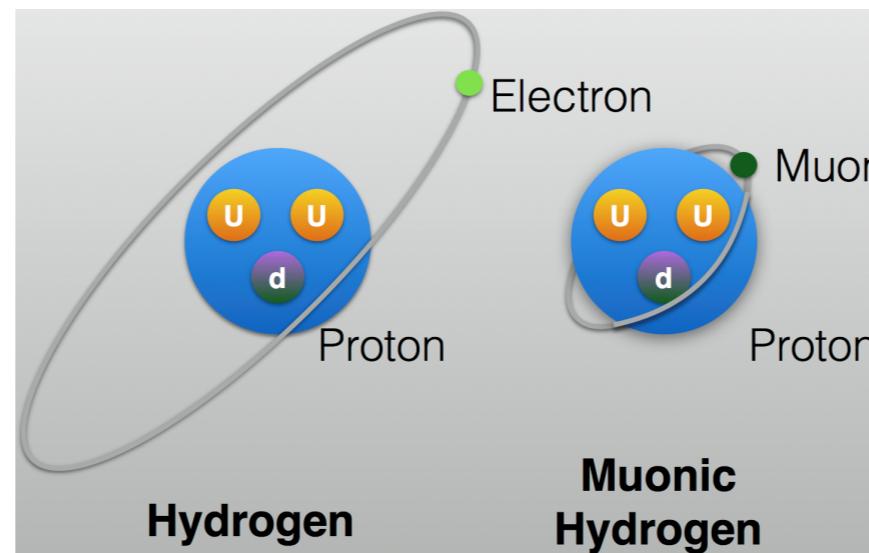
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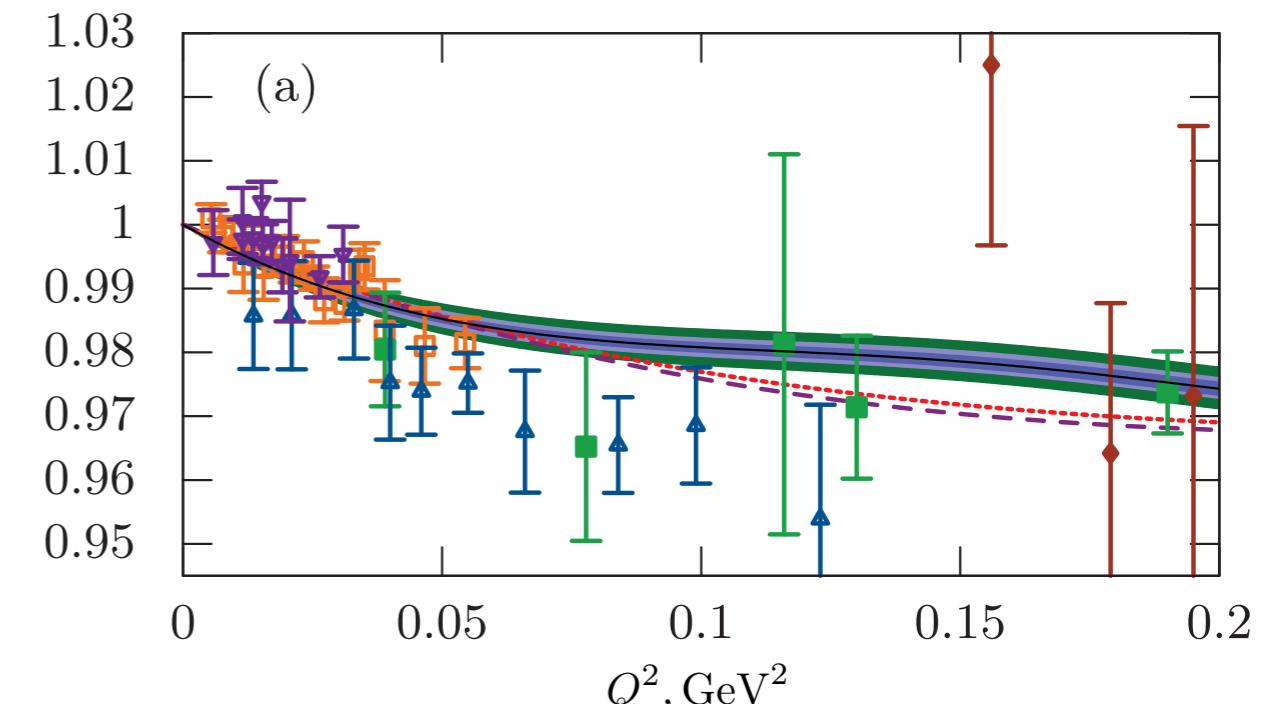
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CREMA (2010, 2013)

eH 2S-4P (Garching, 2017)

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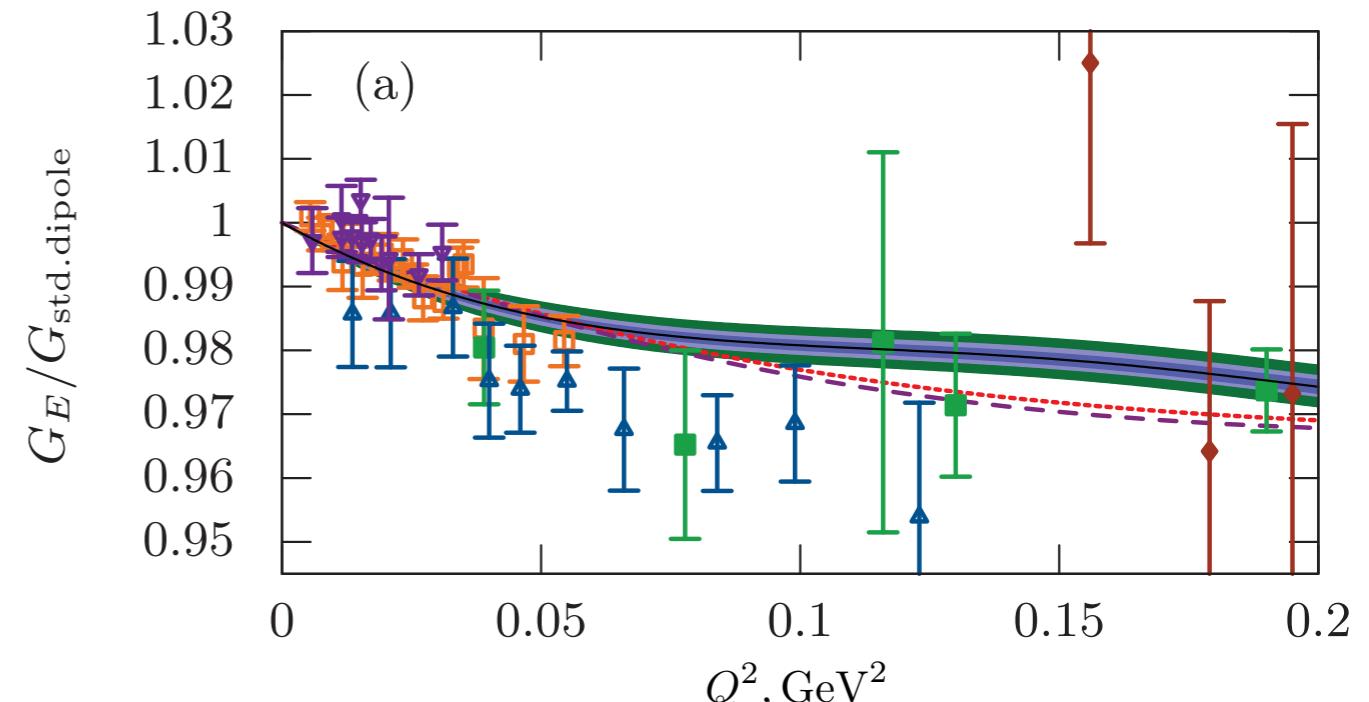
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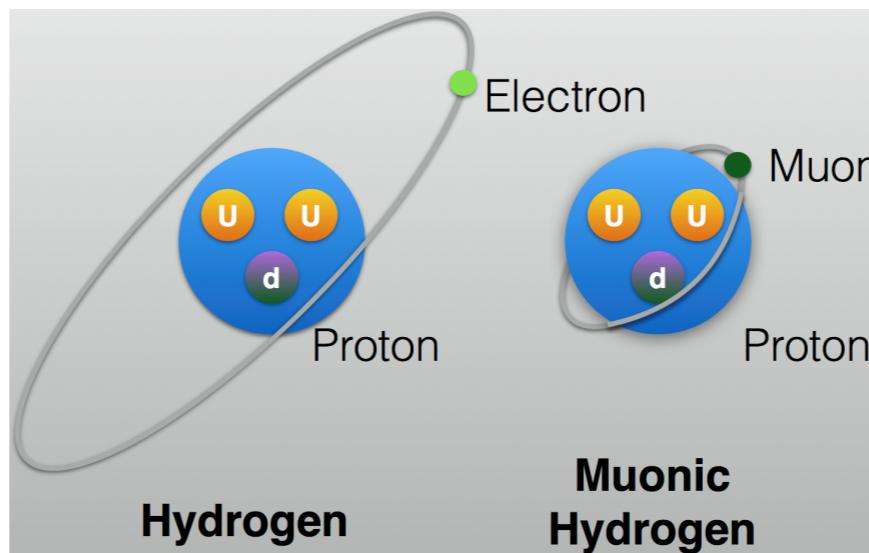
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CODATA 2010

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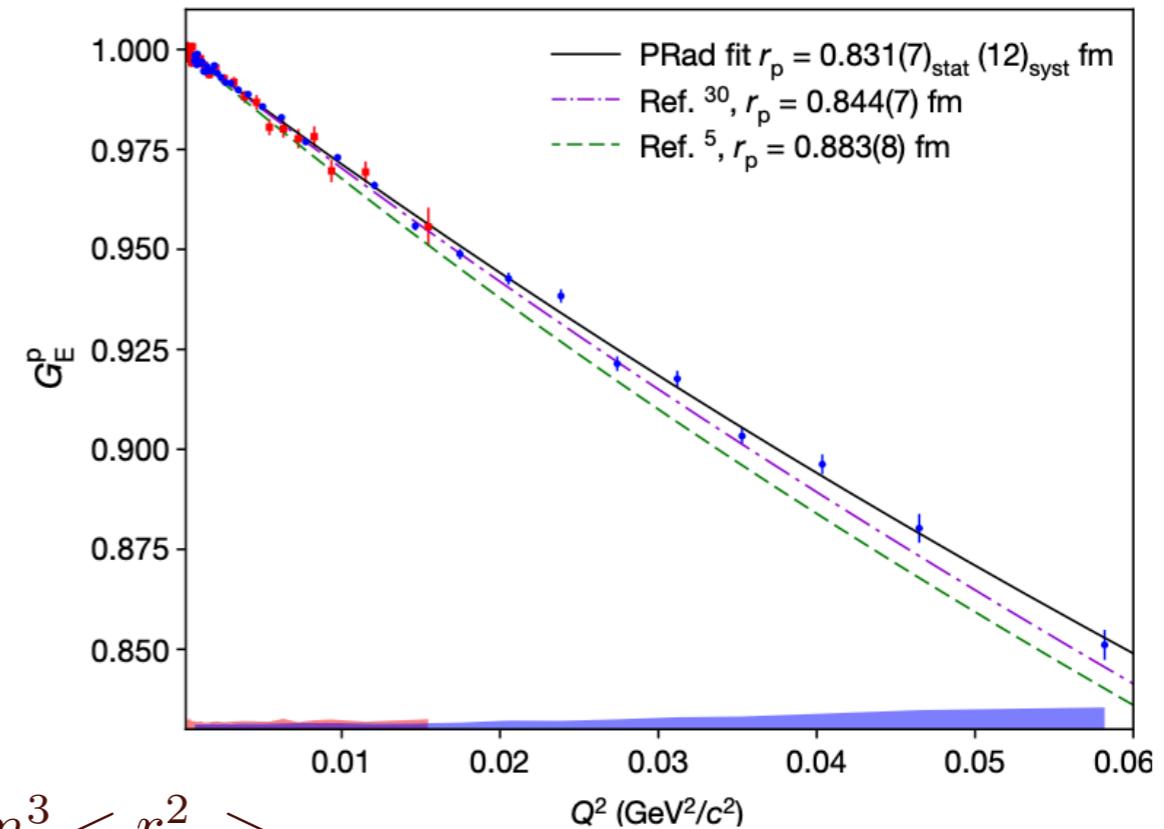
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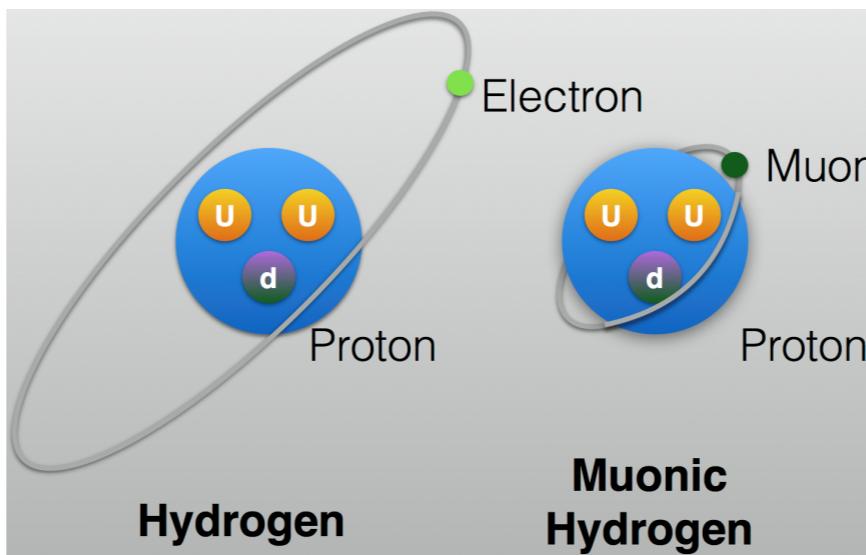
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CODATA 2010

eH 1S-3S (LKB, Paris, 2018)



PRad (JLab, 2019)



μ H, μ D Lamb shift

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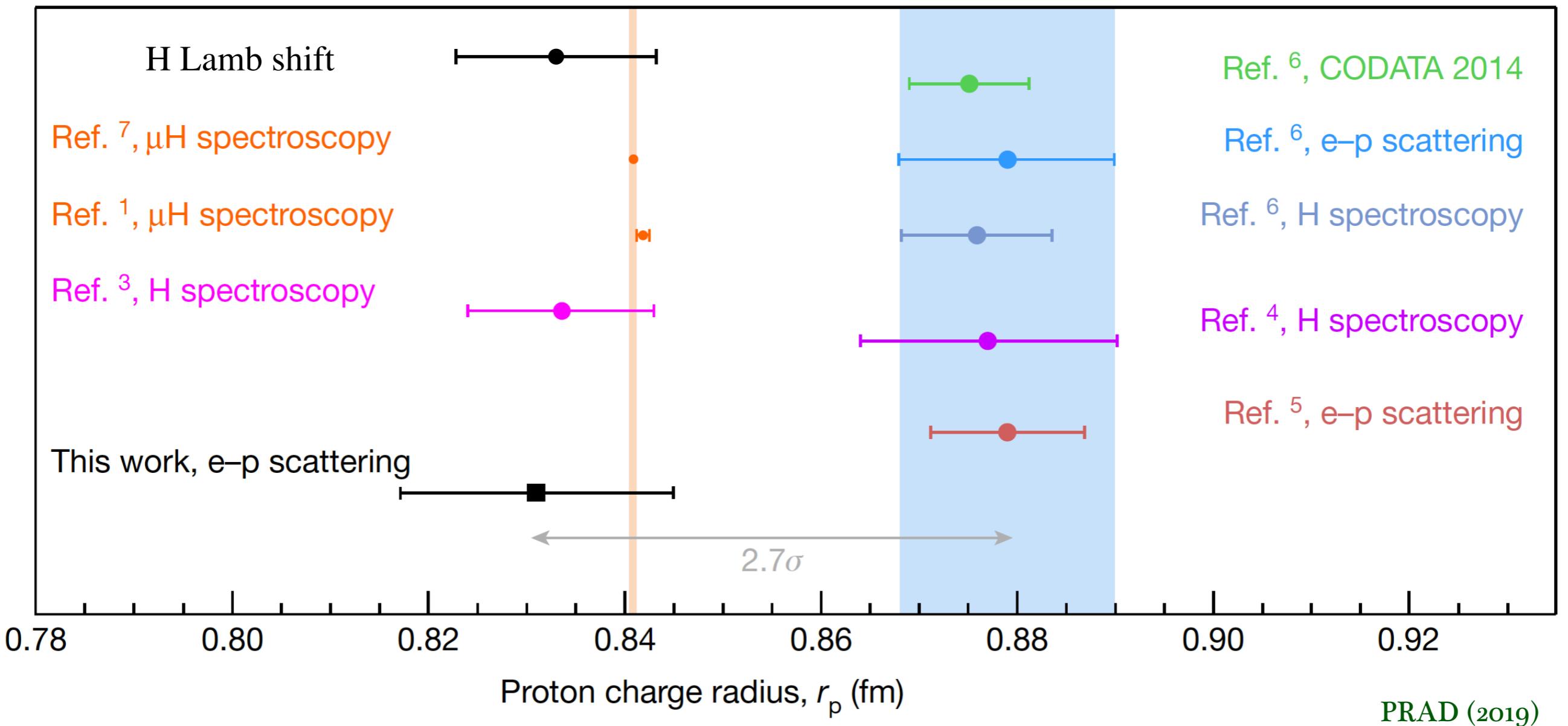
CREMA (2010, 2013)

eH 2S-4P (Garching, 2017)

eH 2S-2P (York U., 2019)

data prefers smaller radius

Proton radius puzzle



- no puzzle in atomic spectroscopy !!!
- scattering data is not completely understood

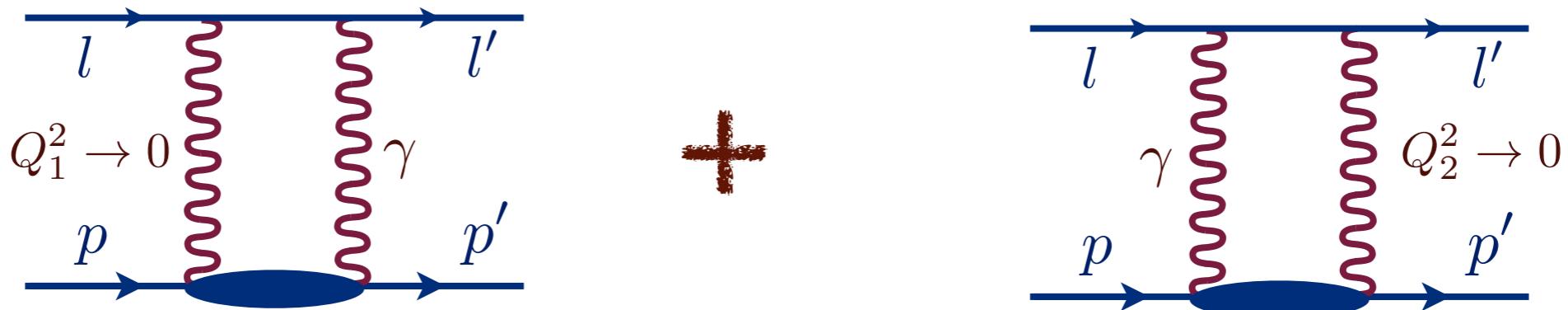
Progress in radiative corrections

Radiative corrections in ep scattering

- standard QED radiative corrections on electron line are known
- proton vertex and hadronic vacuum pol. inside the error budget
- 2γ is not among standard radiative corrections

$$\sigma^{\text{exp}} \equiv \sigma_{1\gamma}(1 + \delta_{\text{rad}} + \delta_{\text{soft}} + \delta_{2\gamma})$$

- soft-photon contribution is included



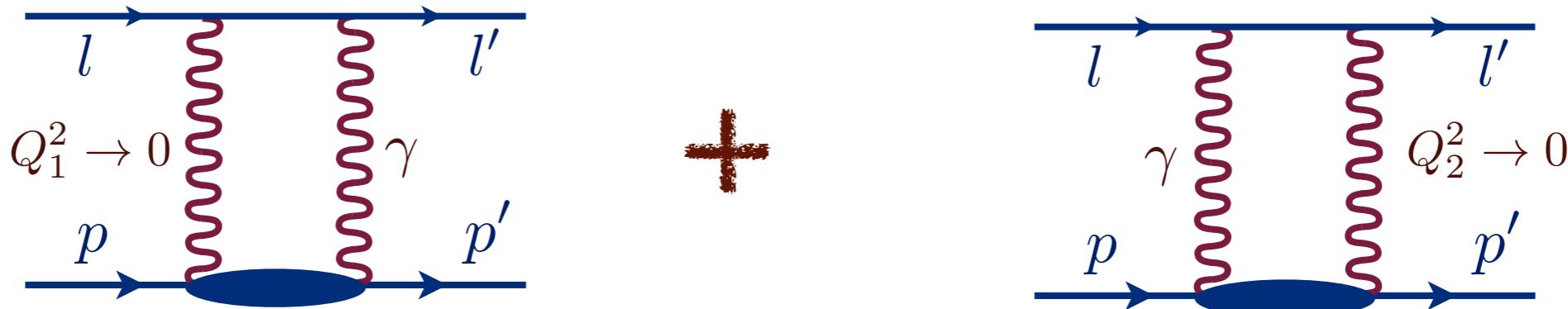
L.C. Maximon and J. A. Tjon (2000)

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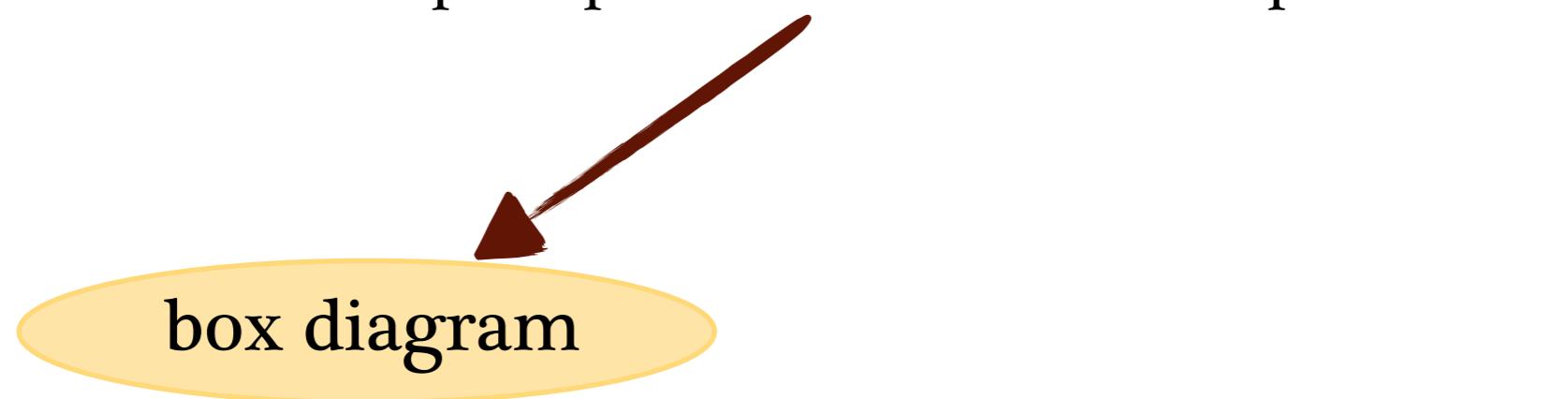
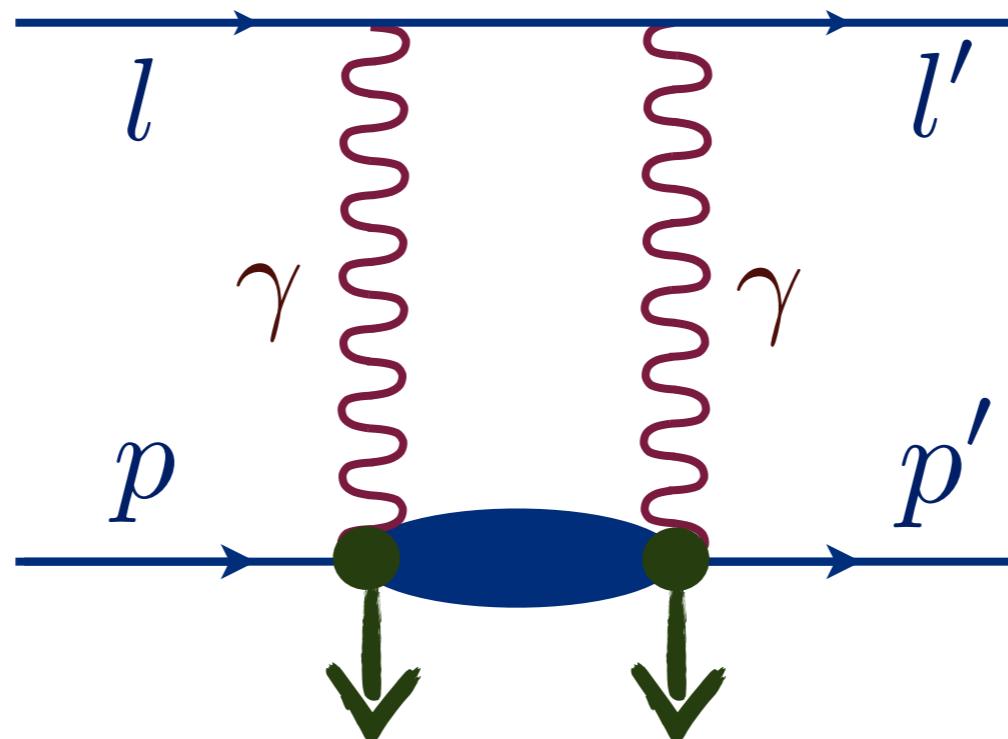
- models for hard-photon contribution

L.C. Maximon and J. A. Tjon (2000)

Feshbach correction: ultrarelativistic scattering in Coulomb field
hadronic model: bulk of proton intermediate state contribution

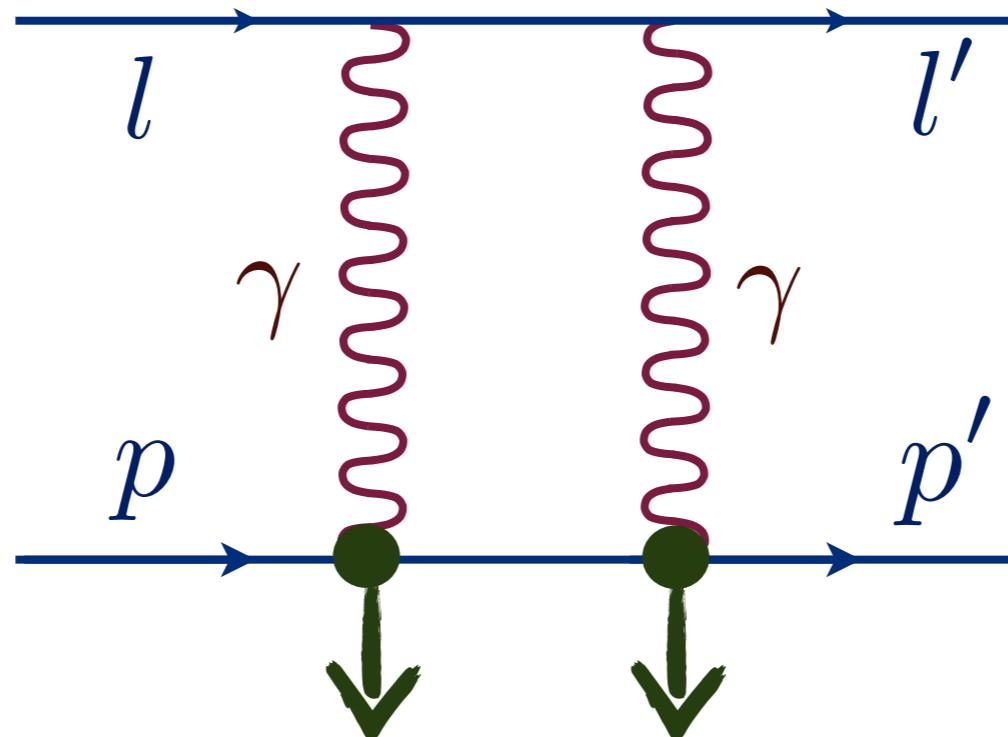
- 2γ contribution requires theoretical studies

non-forward scattering
at low momentum transfer

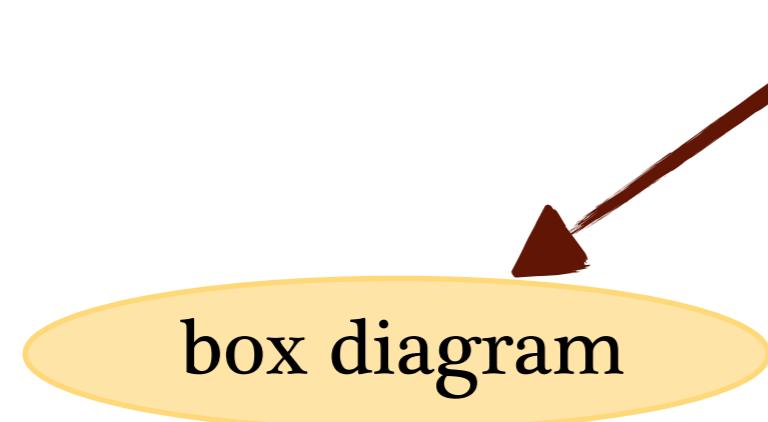


assumption about the vertex

non-forward scattering
proton state



Dirac and Pauli form factors



assumption about the vertex

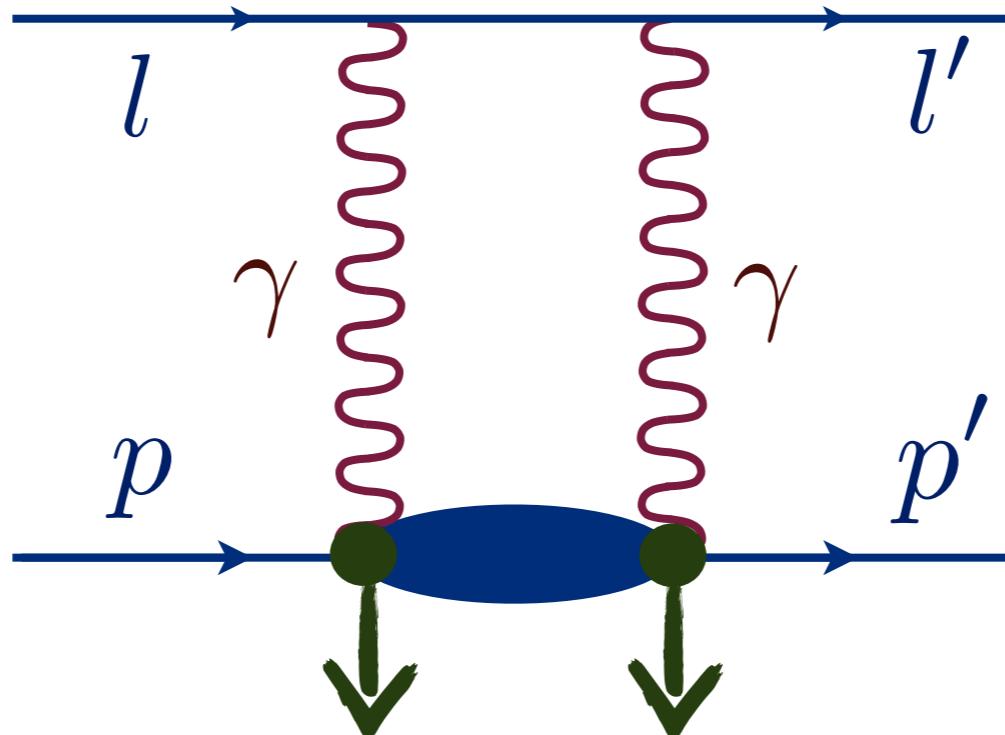
$$\Gamma^\mu(Q^2) = \gamma^\mu F_D(Q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2M} F_P(Q^2)$$

ep scattering: P. G. Blunden, W. Melnitchouk and J. A. Tjon (2003)

violation of unitarity for resonances !

non-forward scattering inelastic states

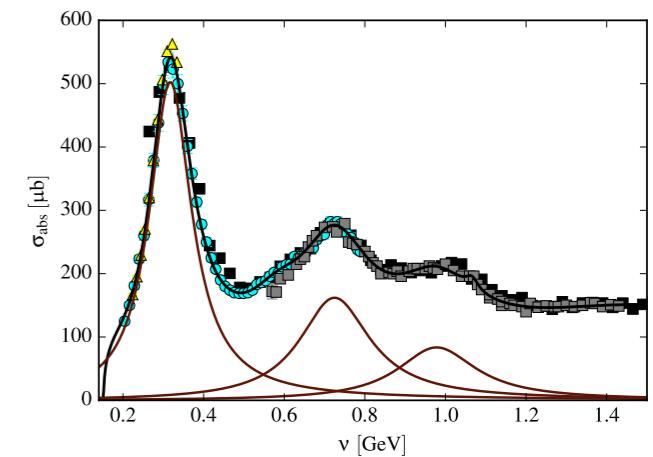
works at small
scattering angles



forward doubly-virtual Compton tensor

box diagram

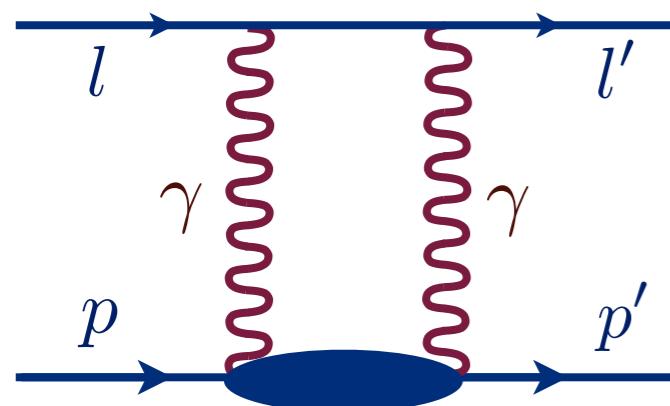
proton + inelastic = total



unpolarized proton structure

M. E. Christy, P. E. Bosted (2010)
JLab data

Low- Q^2 inelastic 2γ correction (e-p)



- 2γ blob: near-forward virtual Compton scattering

Feshbach inelastic elastic

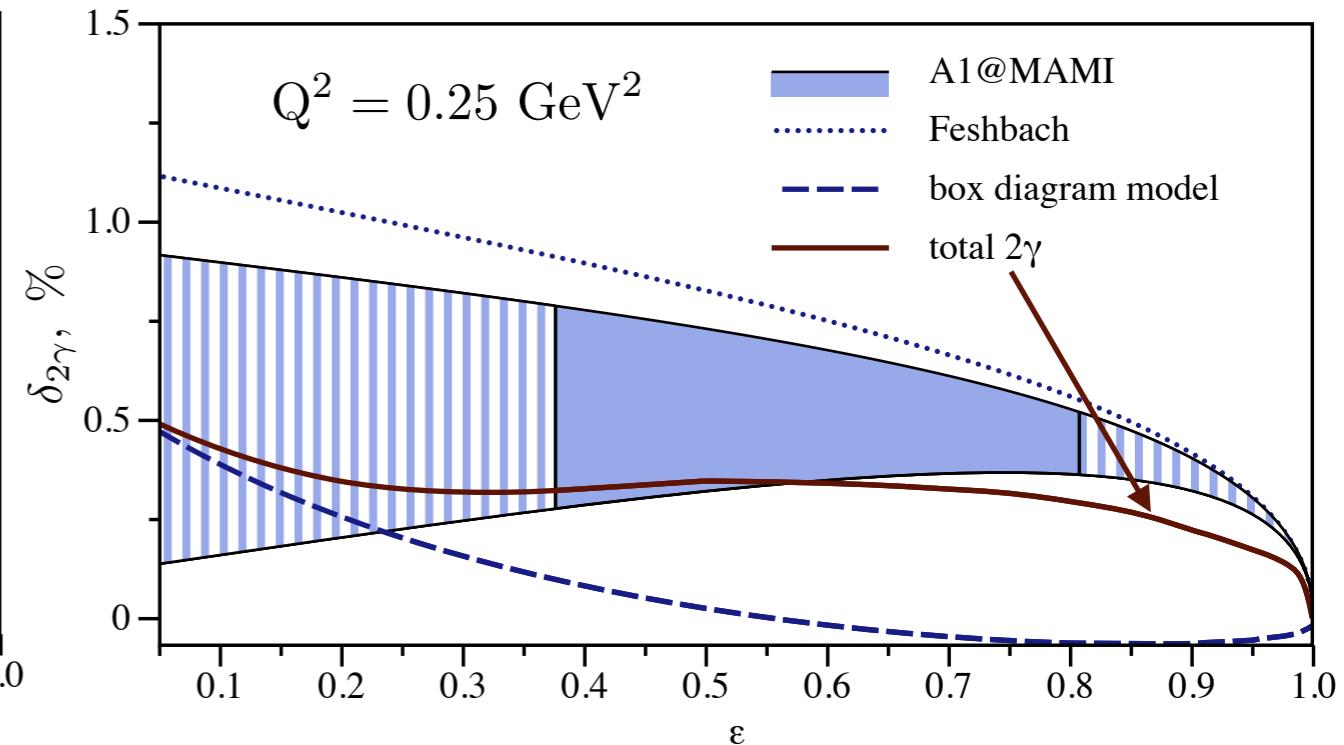
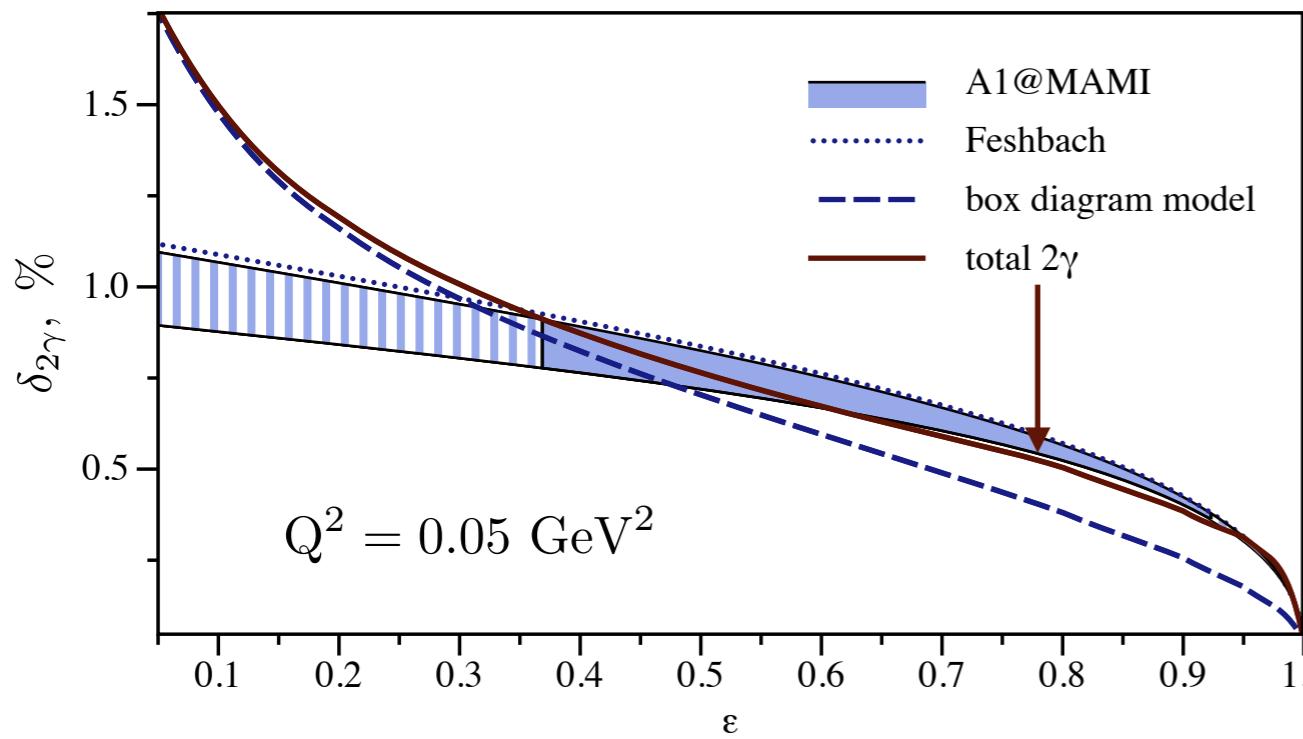
$$ep : \delta_{2\gamma} \sim a \sqrt{Q^2} + b Q^2 \ln Q^2 + c Q^2 \ln^2 Q^2$$

R. W. Brown (1970)

unpolarized proton structure

$$\delta_{2\gamma} = \int d\nu_\gamma dQ^2 (w_1(\nu_\gamma, Q^2) \cdot F_1(\nu_\gamma, Q^2) + w_2(\nu_\gamma, Q^2) \cdot F_2(\nu_\gamma, Q^2))$$

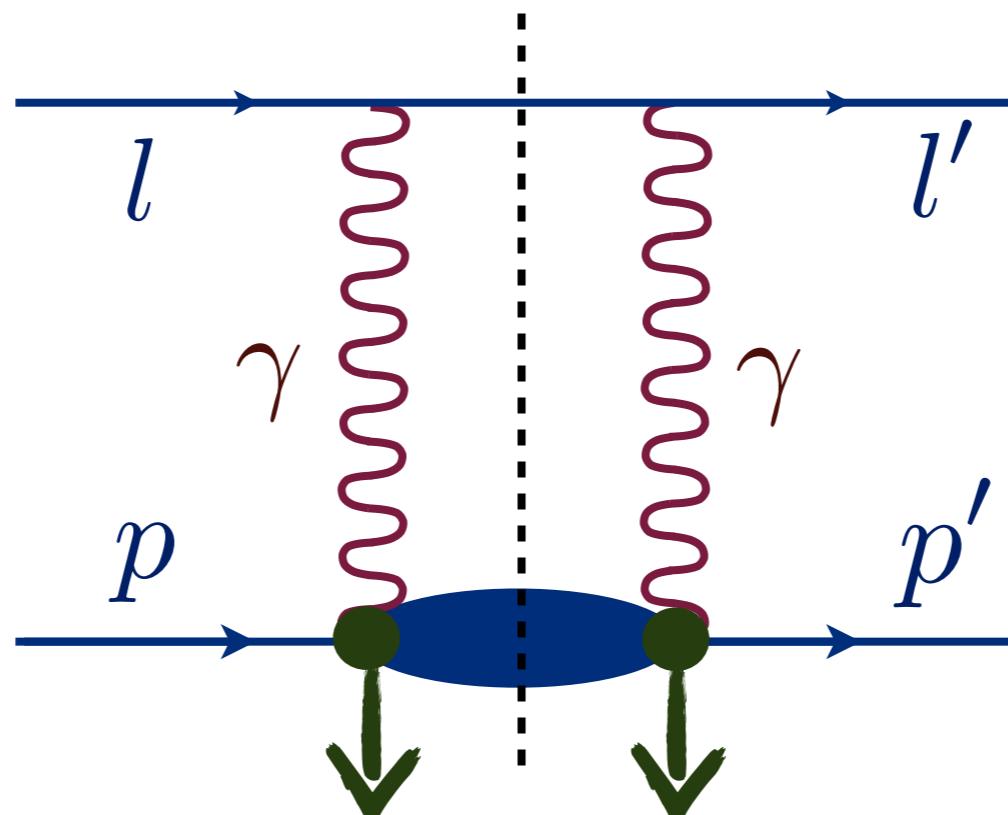
M. E. Christy, P. E. Bosted (2010)



- 2γ at large ϵ agrees with empirical fit

O. T. and M. Vanderhaeghen (2016)

non-forward scattering
at low momentum transfer



photoproduction vertex or Compton tensor

box diagram

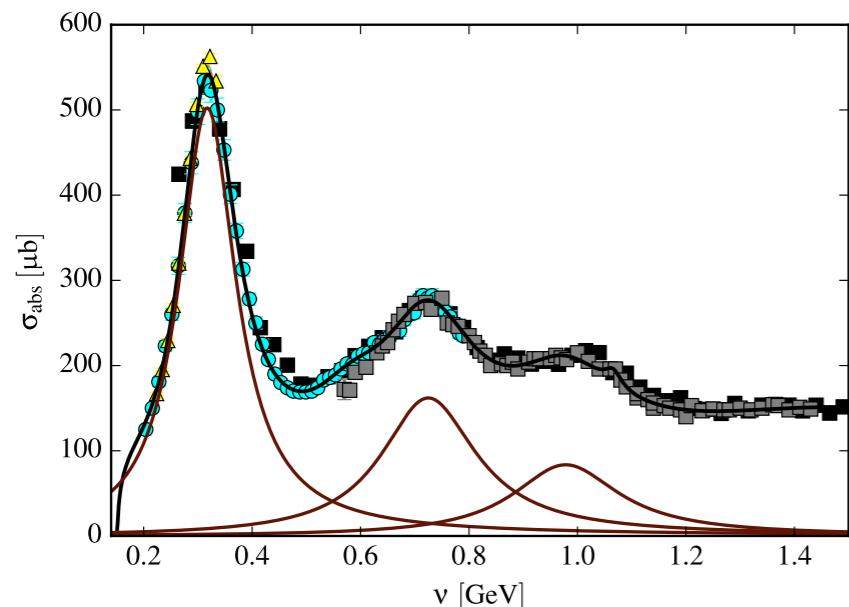
dispersion relations

assumption about the vertex

based on **on-shell** information

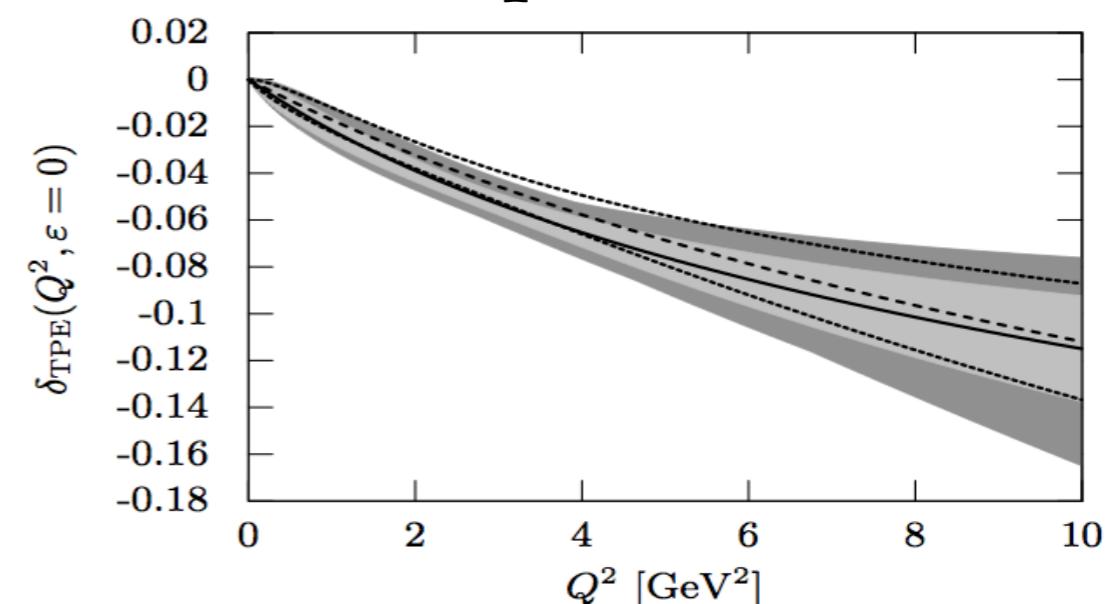
Fixed- Q^2 dispersion relation framework

on-shell 1γ amplitudes



experimental data

2γ prediction



elastic and πN

cross section correction

unitarity



$$\Re \mathcal{F}(\nu) = \frac{2\nu}{\pi} \mathcal{P} \int_{\nu_{min}}^{\infty} \frac{\Im \mathcal{F}(\nu' + i0)}{\nu'^2 - \nu^2} d\nu'$$

2γ imaginary parts

disp. rel.

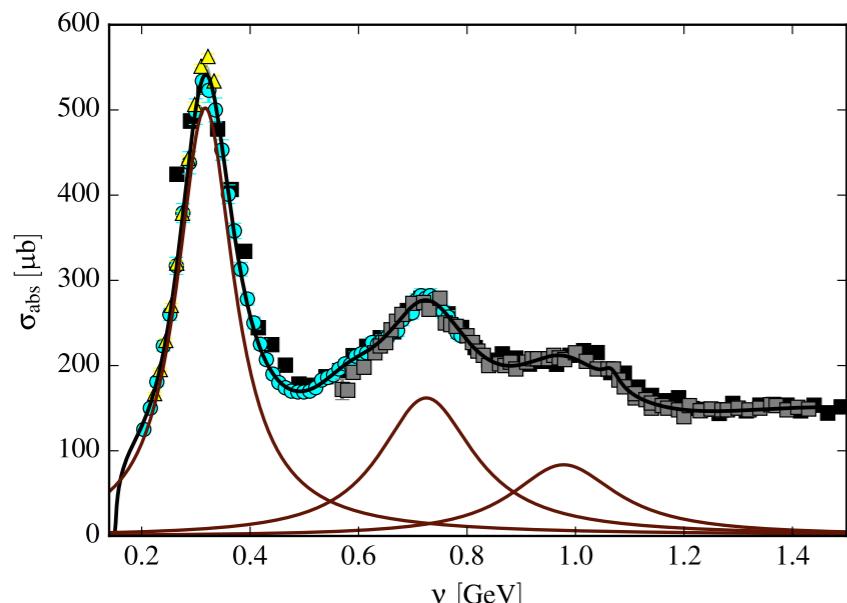


2γ real parts



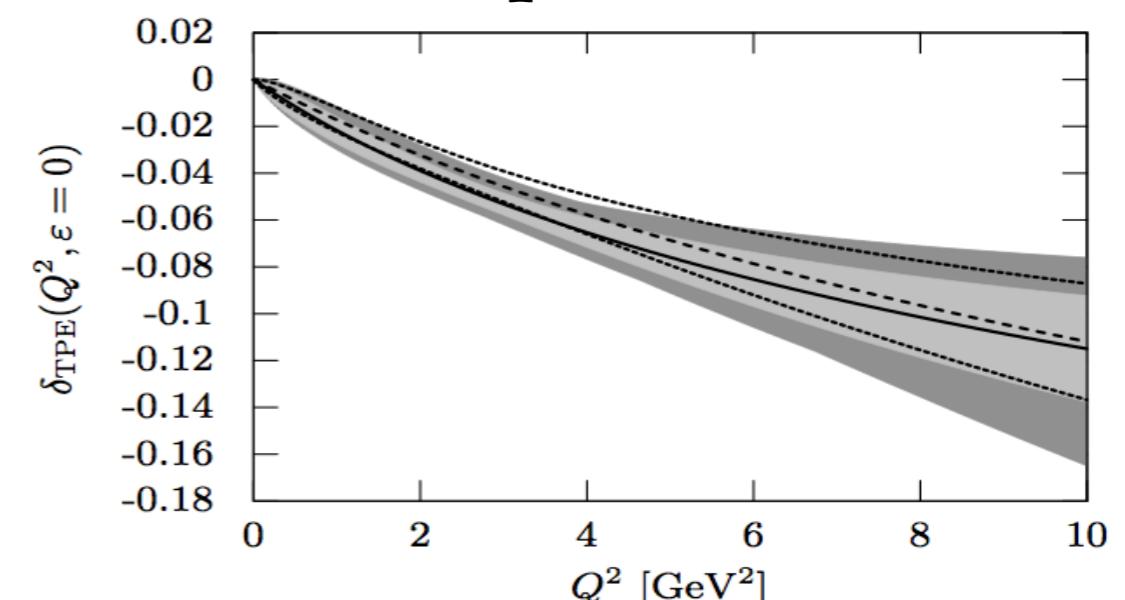
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experimental data

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elastic and πN

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MAID2007 eN->eπN amplitudes

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$$\Re \mathcal{F}(\nu) = \frac{2\nu}{\pi} \mathcal{P} \int_{\nu_{min}}^{\infty} \frac{\Im \mathcal{F}(\nu' + i0)}{\nu'^2 - \nu^2} d\nu'$$

2 γ imaginary parts

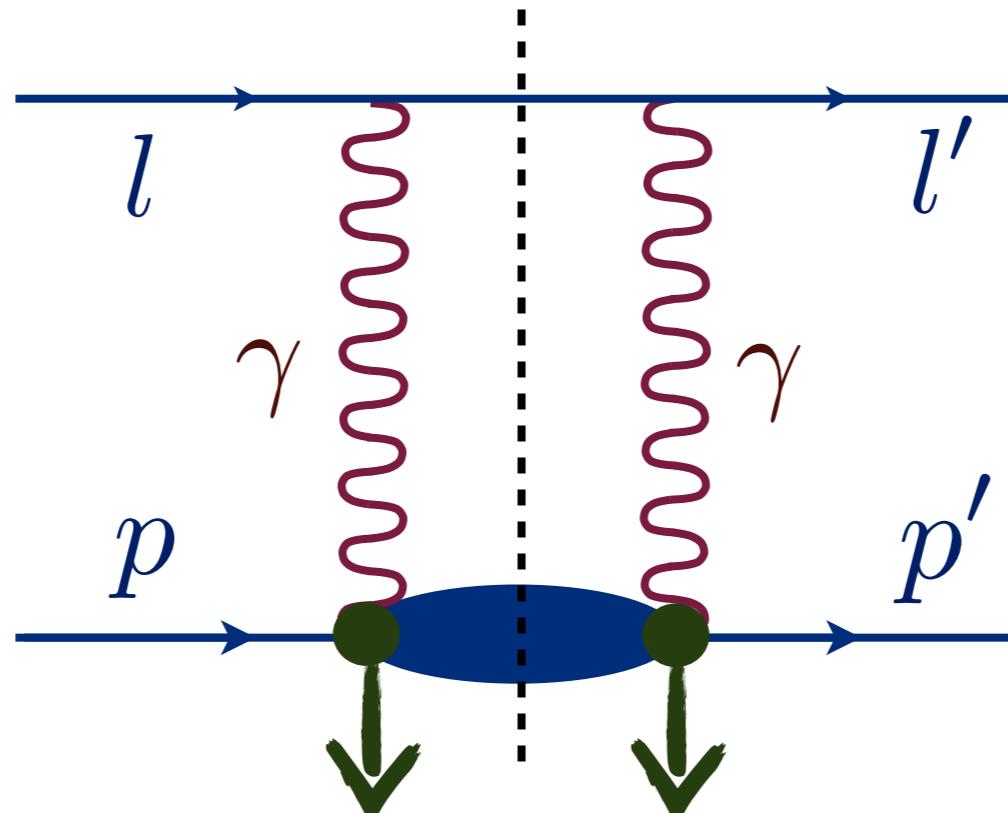
disp. rel.



2 γ real parts



non-forward scattering



photoproduction vertex or Compton tensor

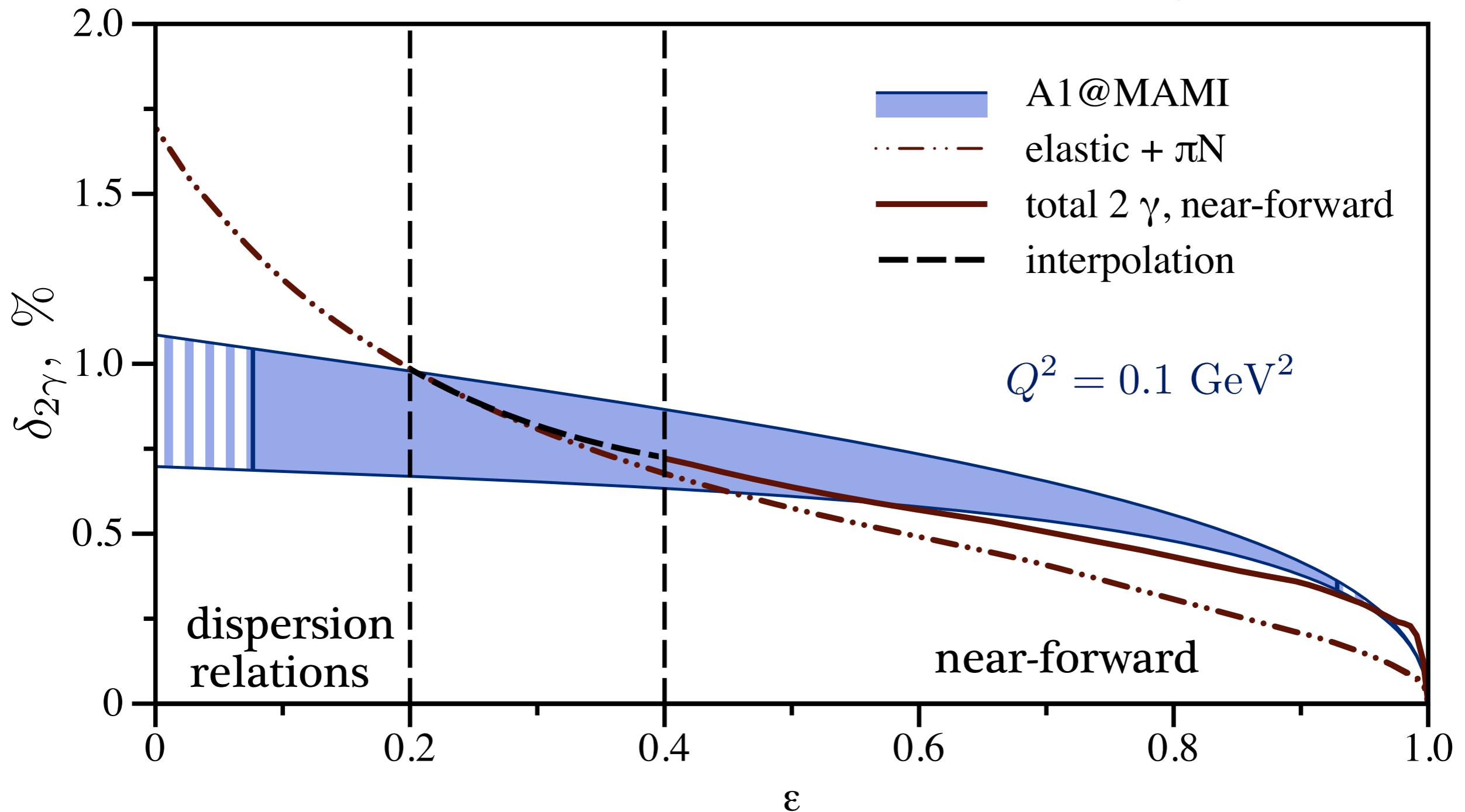
box diagram

dispersion relations

valid at small scattering angles

based on **on-shell** information

Our best 2γ knowledge

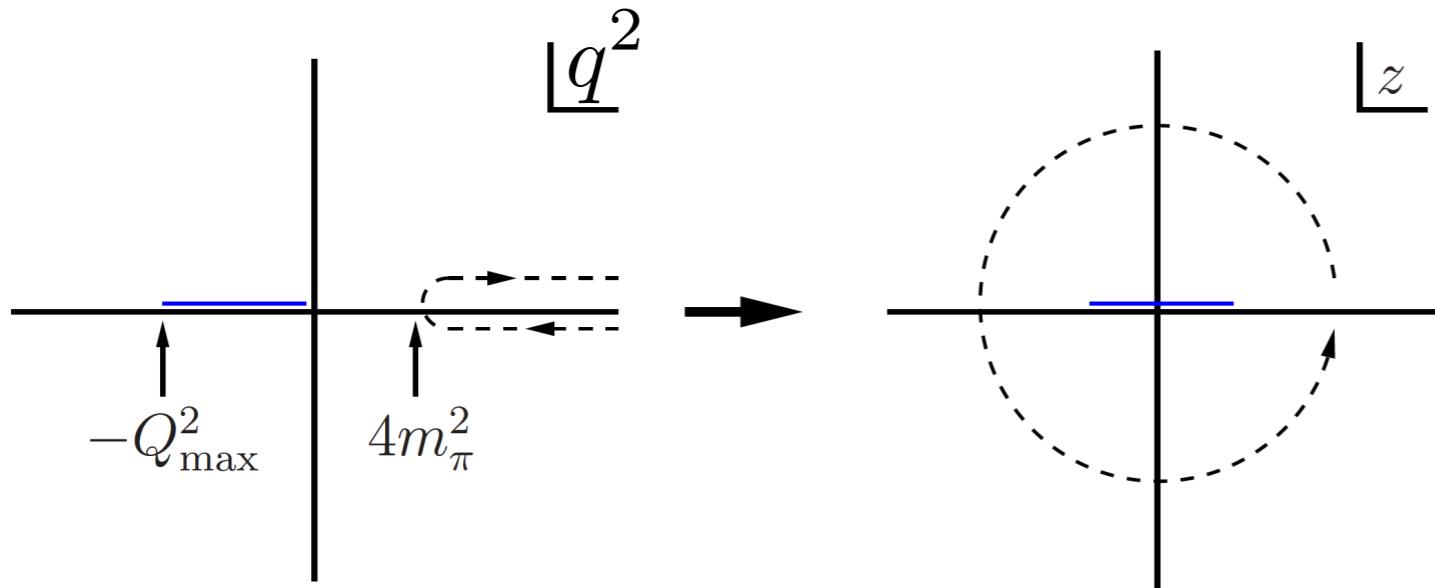


- small Q^2 : near-forward at large ϵ , all inelastic states
- $Q^2 \lesssim 1 \text{ GeV}^2$: elastic+ πN within dispersion relations
- intermediate range: interpolation; one fit parameter for change in 2γ

z-expansion, constraints and results

z-expansion ansatz

- form factors are analytic in Q^2 plane besides two-pion cut



$$z(Q^2) = \frac{\sqrt{t_{cut} + Q^2} - \sqrt{t_{cut} - t_0}}{\sqrt{t_{cut} + Q^2} + \sqrt{t_{cut} - t_0}}$$

- mapping to true expansion in small parameter z

$$G(Q^2) = \sum_{k=0}^{k_{\max}} a_k z(Q^2)^k$$

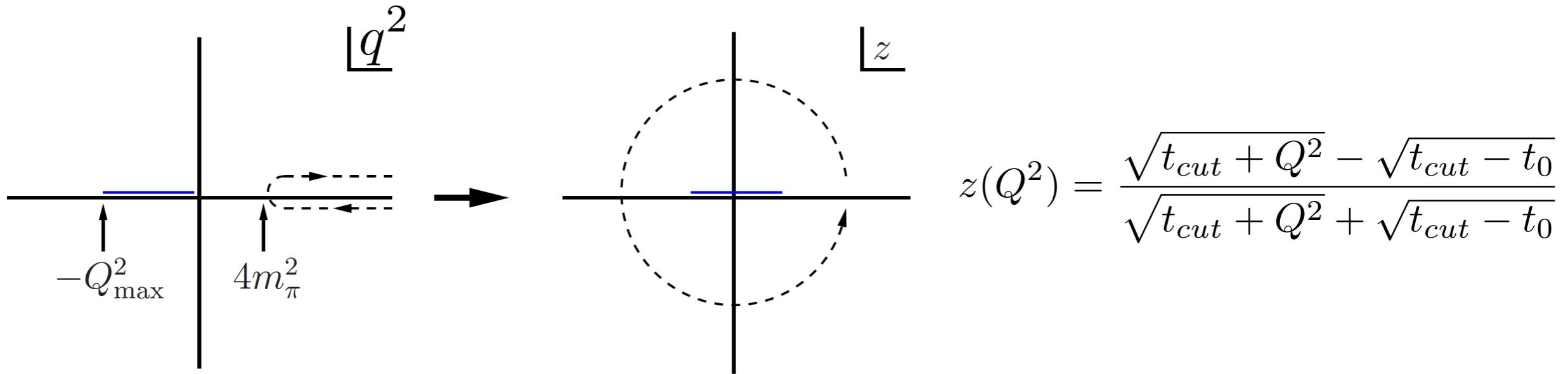
R. J. Hill and G. Paz (2010)

G. Lee, J. Arrington and R. J. Hill (2015)

Z. Ye, J. Arrington, R. J. Hill and G. Lee (2018)

z-expansion ansatz

- form factors are analytic in Q^2 plane besides two-pion cut



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R. J. Hill and G. Paz (2010)

G. Lee, J. Arrington and R. J. Hill (2015)

Z. Ye, J. Arrington, R. J. Hill and G. Lee (2018)

- convergence requires bounds on parameters a_k
- t_0 is chosen to minimize z in the region of available data
- normalization (charge or magnetic moment) is imposed
- pQCD asymptotic behavior $G(Q^2) < 1/Q^3$: four sum rules

S. Brodsky and P. Lepage (1980)

- $k_{max}-4$ free parameters in convergent expansion

Available data and results

- charge radii are treated as data points

$$(r_E^p)_{\mu H} = 0.84087(39) \text{ fm} \quad \langle r^2 \rangle_E^n = -0.1161(22) \text{ fm}$$

Kopecky (1995, 1997)

- to avoid data tensions, Mainz dataset only chosen as default

fit	$Q_{\max}^2 [\text{GeV}^2]$	Mainz	World	Pol	G_E^n	G_M^n	r_E^p	$\langle r^2 \rangle_E^n$	χ^2	n_{dof}
p	1.0	657	0	0	0	0	1	0	475.35	650
$n (G_E^n)$	1.0	0	0	0	29	0	0	1	14.81	26
$n (G_M^n)$	1.0	0	0	0	0	15	0	0	8.03	11
iso (1 GeV^2)	1.0	657	0	0	29	15	1	1	499.63	687
iso (3 GeV^2)	3.0	657	480	58	37	23	1	1	1162.45	1241

- neutron, proton and iso form factors: form of fit for axial

A.S. Meyer, M. Betancourt, R. Gran and R.J. Hill (2016)

- e.g., fit of proton electric form factor:

$$[a_1^p, a_2^p, a_3^p, a_4^p] = [-1.4860(97), -0.096(52), 1.82(15), 1.29(41)]$$

- covariance matrices and fits with $k_{\max} \rightarrow k_{\max} + 1$ for error estimates

- 4-parameter representation of form factors with errors and correlations

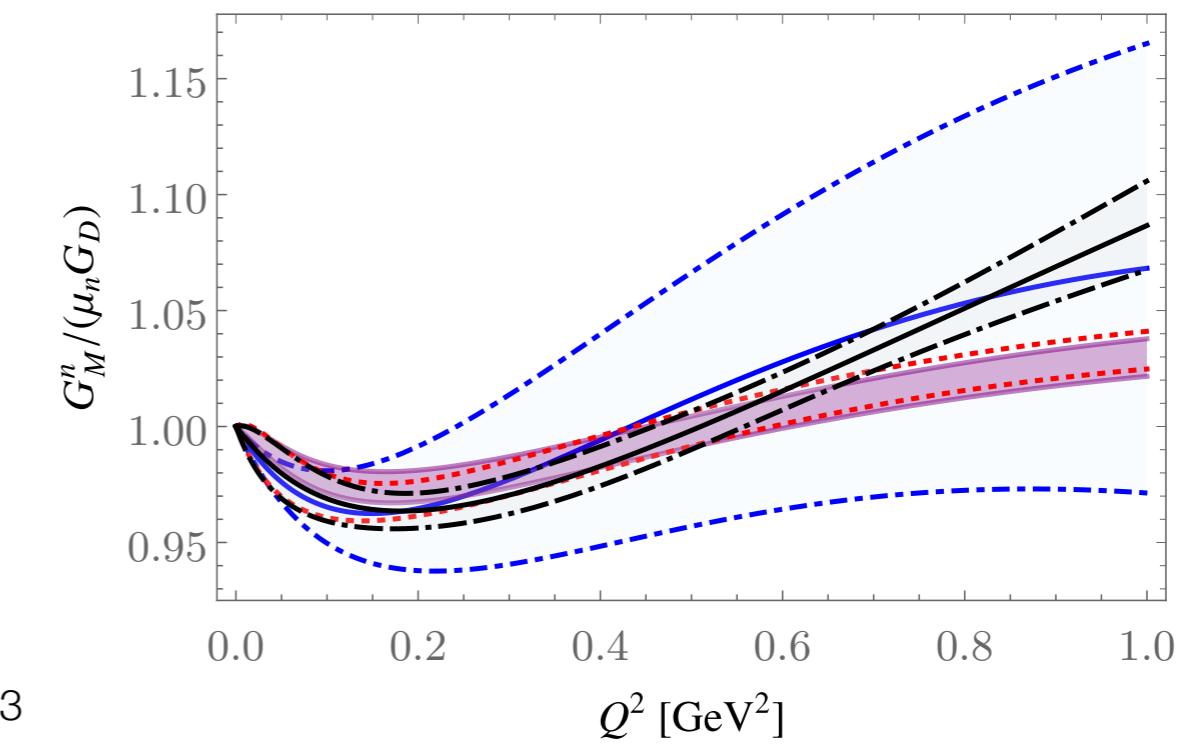
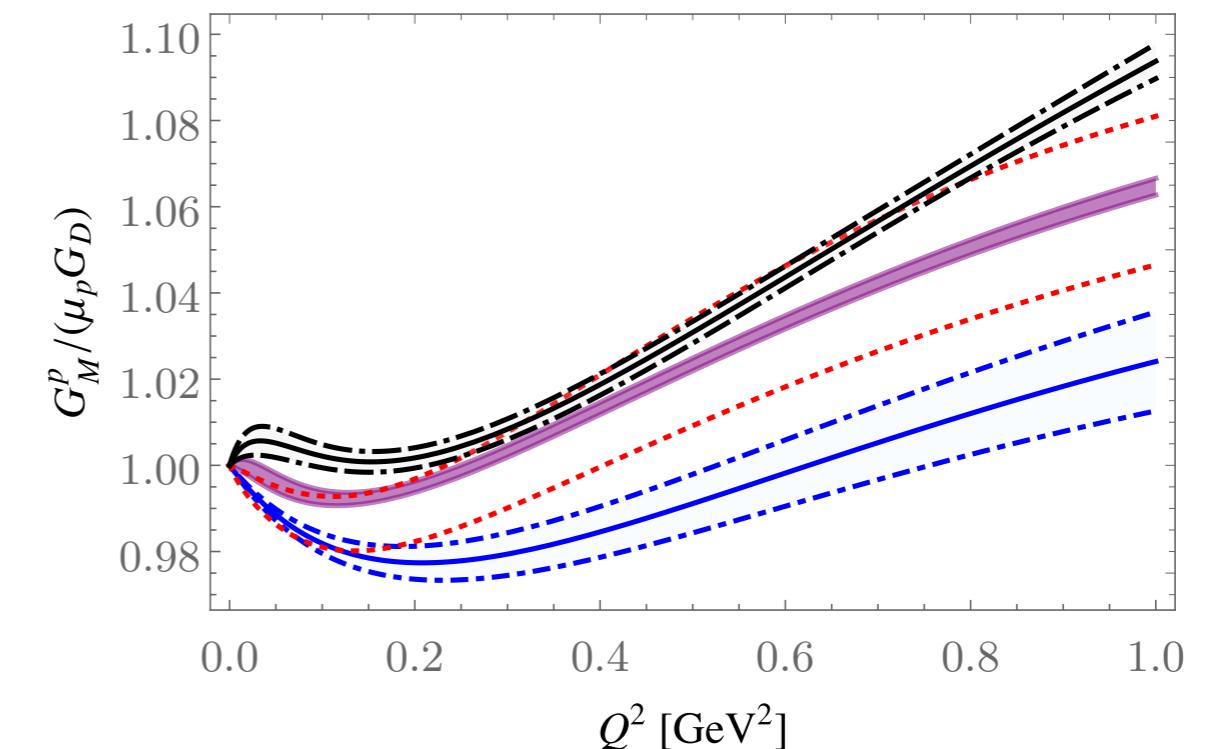
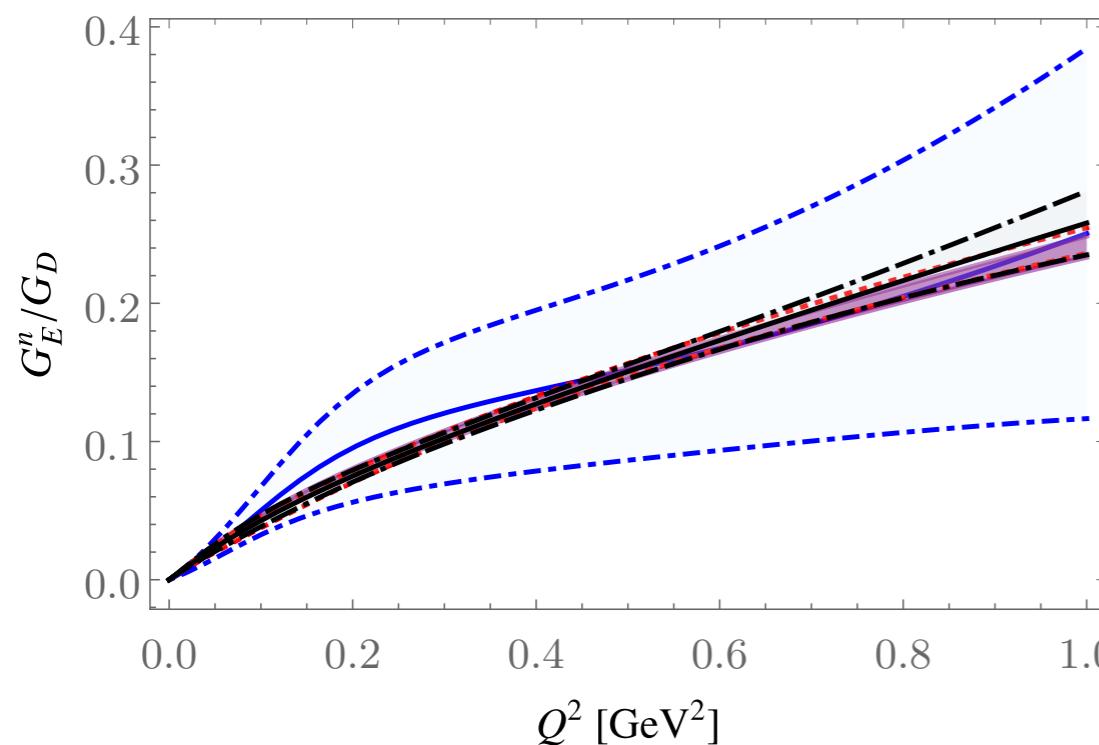
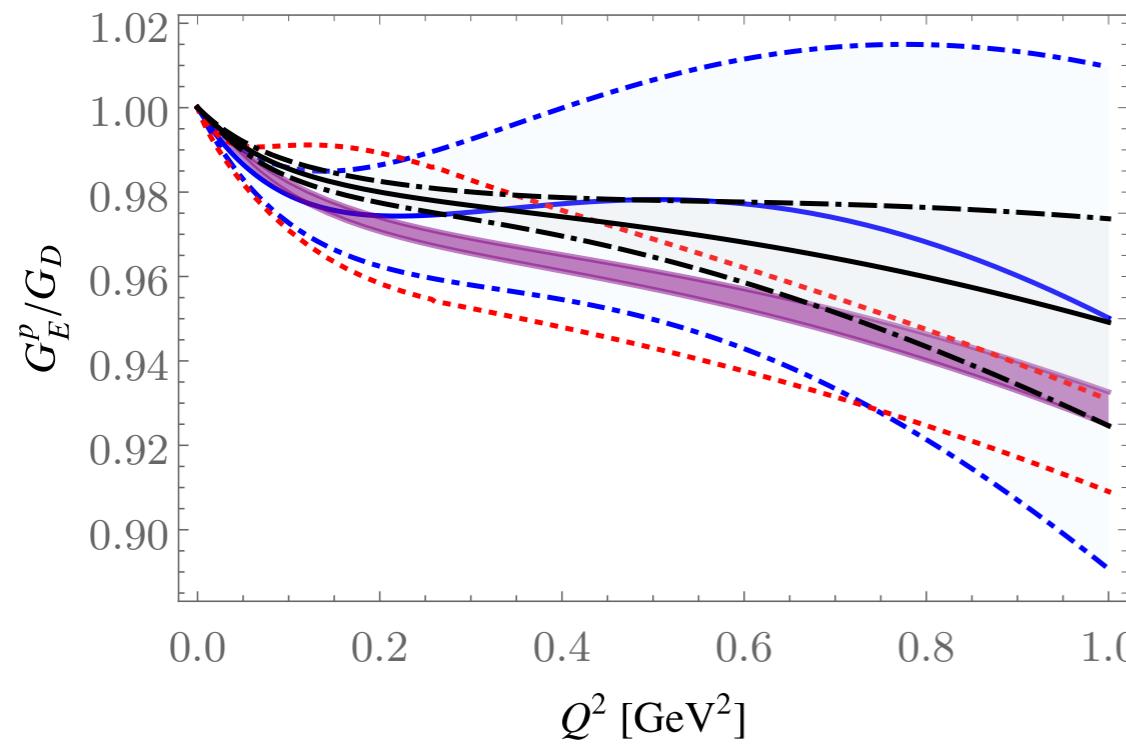
Nucleon electromagnetic form factors

- fit normalized to dipole form

$$G_D(Q^2) = \frac{G_D(0)}{(1 + Q^2/\Lambda^2)^2}$$

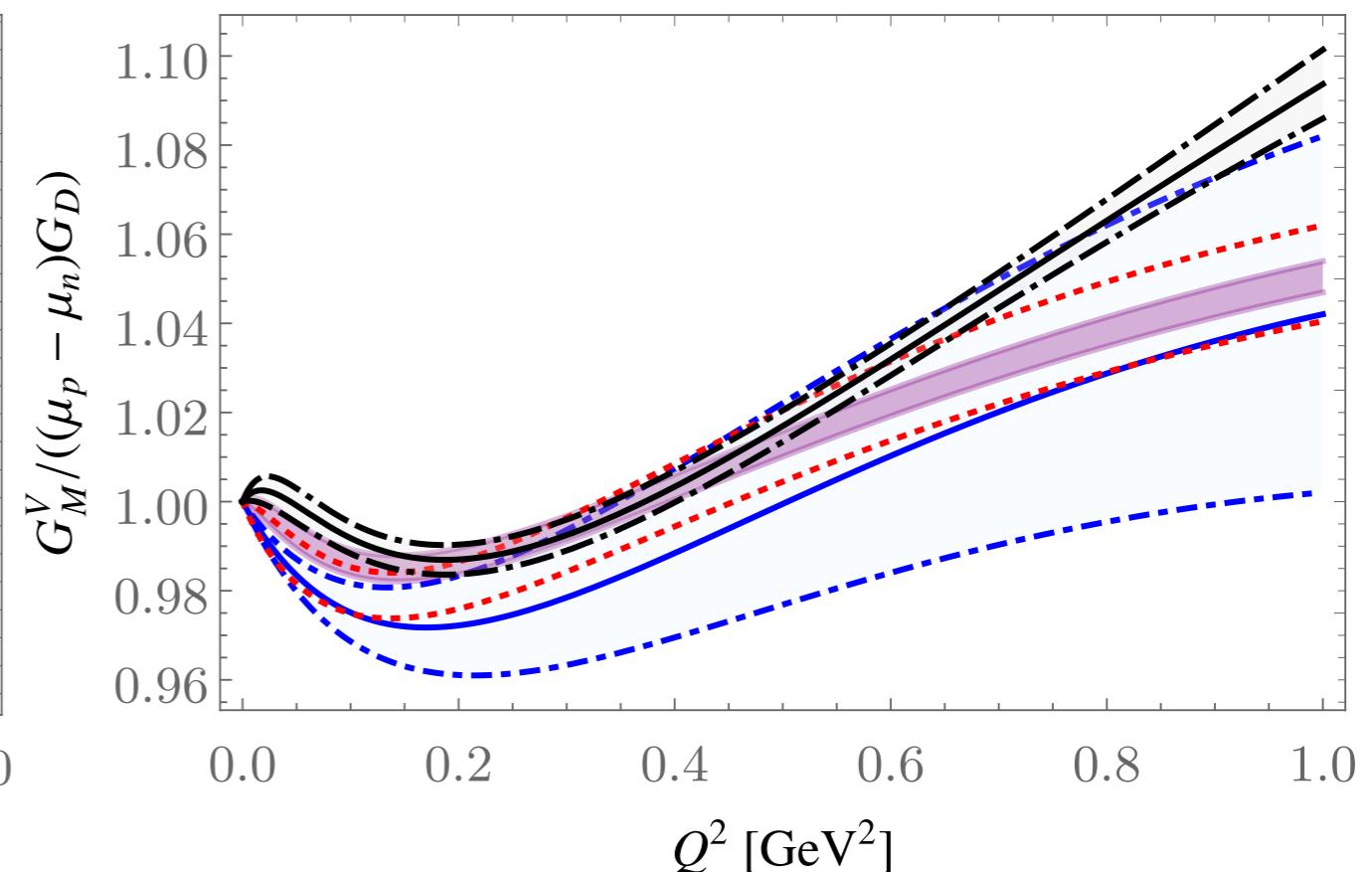
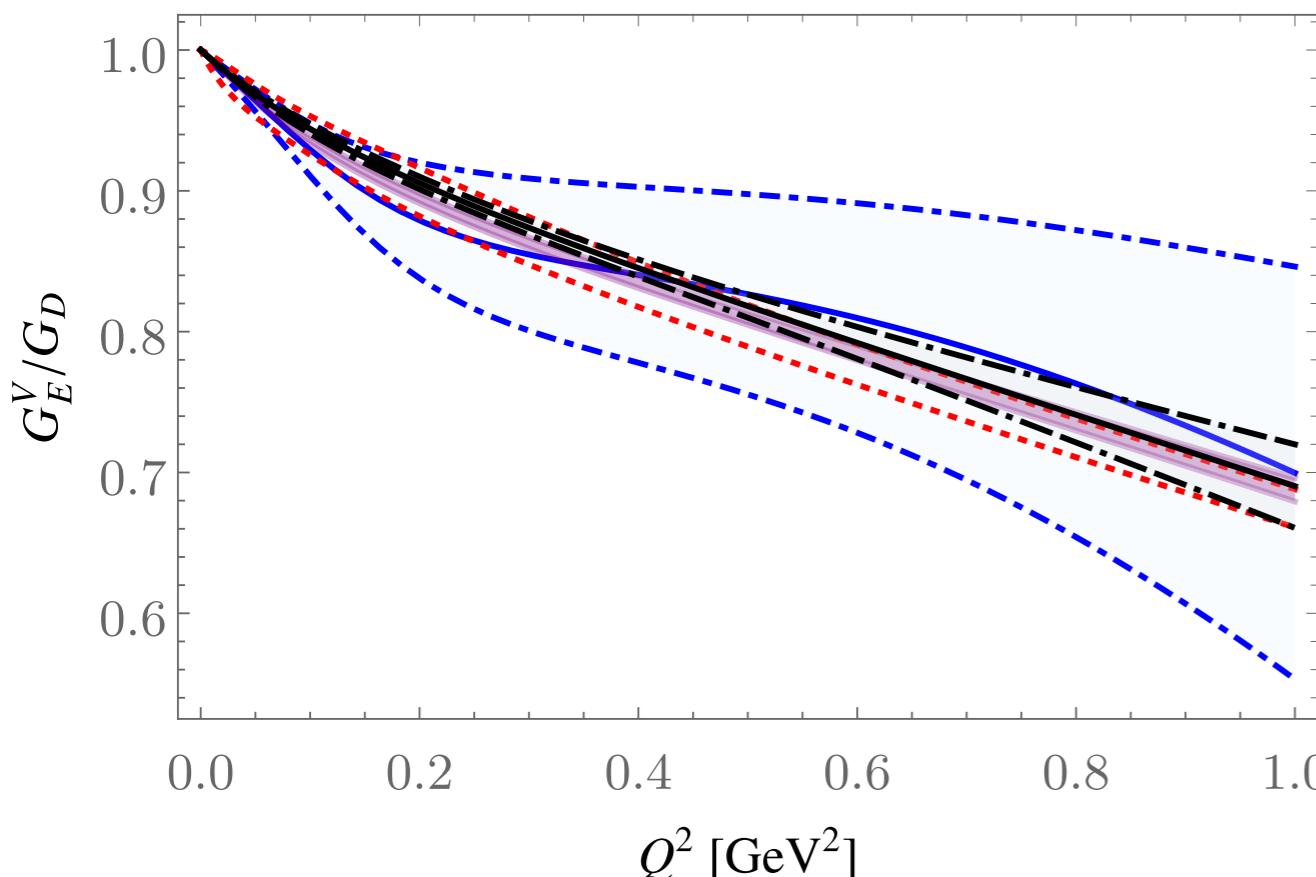
$$\Lambda^2 = 0.71 \text{ GeV}^2$$

- iso 1, **BBBA2005**, iso 3, **global fit of Ye et al. (2017)**



Isovector vector form factors

- fit normalized to dipole form $G_D(Q^2) = \frac{G_D(0)}{(1 + Q^2/\Lambda^2)^2}$ $\Lambda^2 = 0.71 \text{ GeV}^2$
- iso 1, **BBBA2005**, iso 3, **global fit of Ye et al. (2017)**



$$G_{E,M}^V = G_{E,M}^p - G_{E,M}^n$$

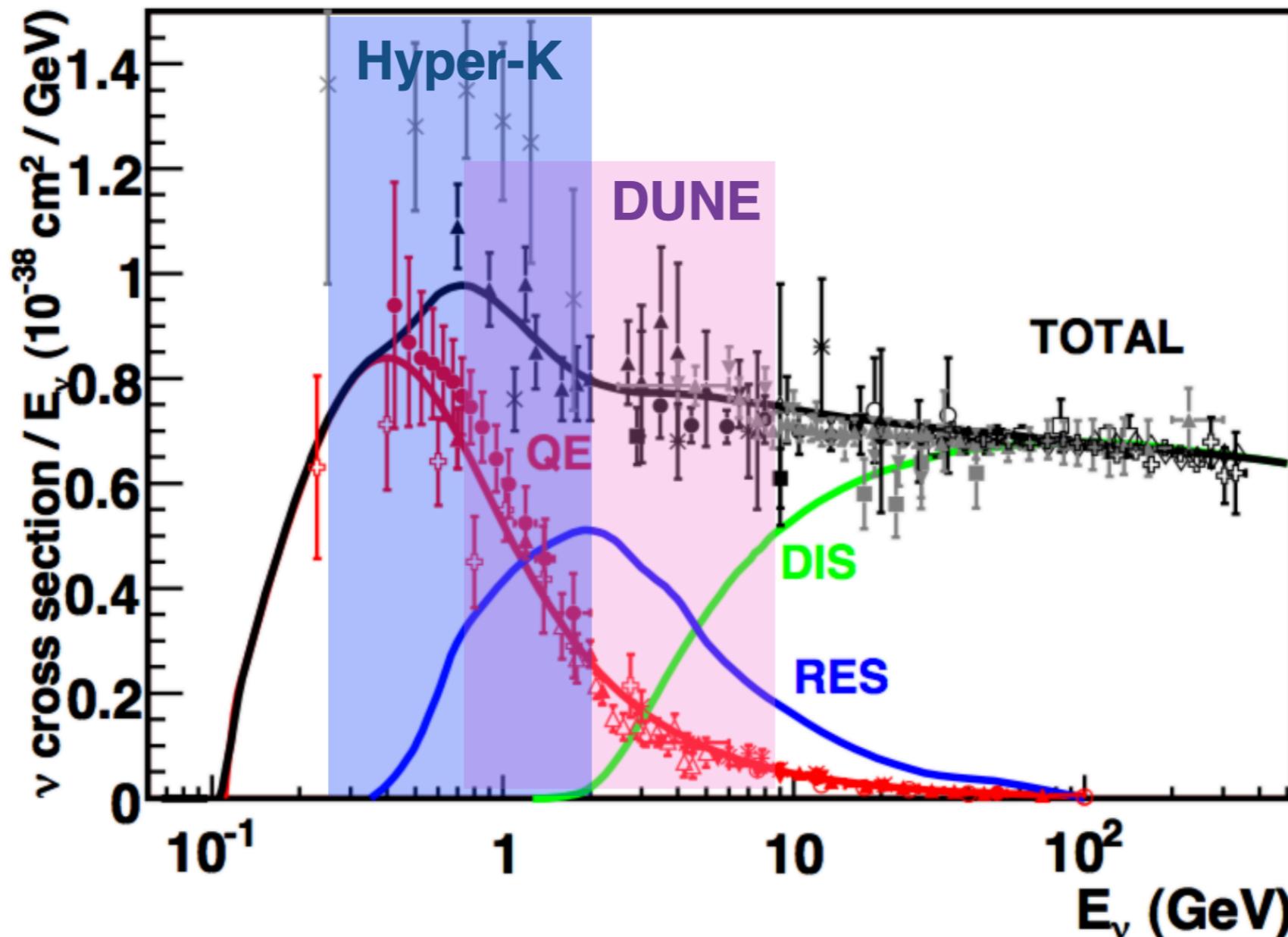


(Anti)neutrino-nucleon charged-current quasielastic scattering



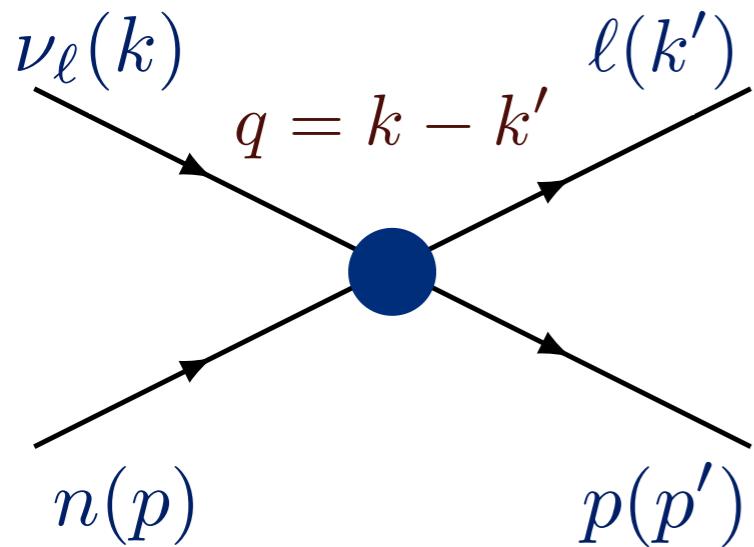
CCQE. Why should we care?

- neutrino-nucleus cross sections and future accelerator-based fluxes



- basic process: bulk of events at Hyper-K and DUNE
- channel for reconstruction of neutrino energy

CCQE scattering on free nucleon



neutrino energy

$$E_\nu$$

momentum transfer

$$Q^2 = -q^2$$

contact interaction at GeV energies

- assuming isospin symmetry, nucleon current:

$$\Gamma^\mu(Q^2) = \gamma^\mu F_D^V(Q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2M} F_P^V(Q^2) + \gamma^\mu \gamma_5 F_A(Q^2) + \frac{q^\mu}{M} \gamma_5 F_P(Q^2)$$

form factors: isovector Dirac and Pauli

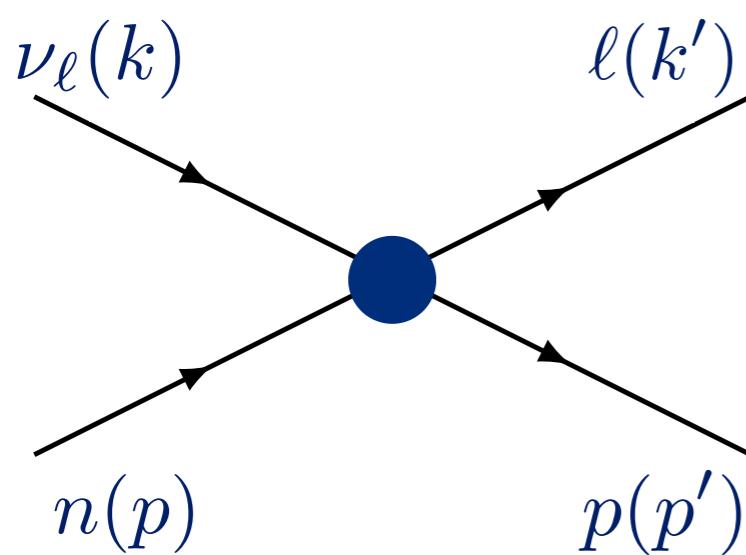
axial and pseudoscalar

$$F_{D,P}^V = F_{D,P}^p - F_{D,P}^n$$

tree-level amplitude

$$T = \frac{G_F V_{ud}}{\sqrt{2}} (\bar{\ell}(k') \gamma_\mu (1 - \gamma_5) \nu_\ell(k)) (\bar{p}(p') \Gamma^\mu(Q^2) n(p))$$

CCQE scattering on free nucleon



$$s - u = 4ME_\nu - Q^2 - m_\ell^2$$
$$\tau = \frac{Q^2}{4M^2}$$

unpolarised observables are measured
cross section expression

$$\frac{d\sigma}{dQ^2} \sim \frac{M^2}{E_\nu^2} \left(A(Q^2) \frac{m_\ell^2 + Q^2}{M^2} - B(Q^2) \frac{s - u}{M^2} + C(Q^2) \left(\frac{s - u}{M^2} \right)^2 \right)$$

Llewellyn Smith

- structure-dependent functions:

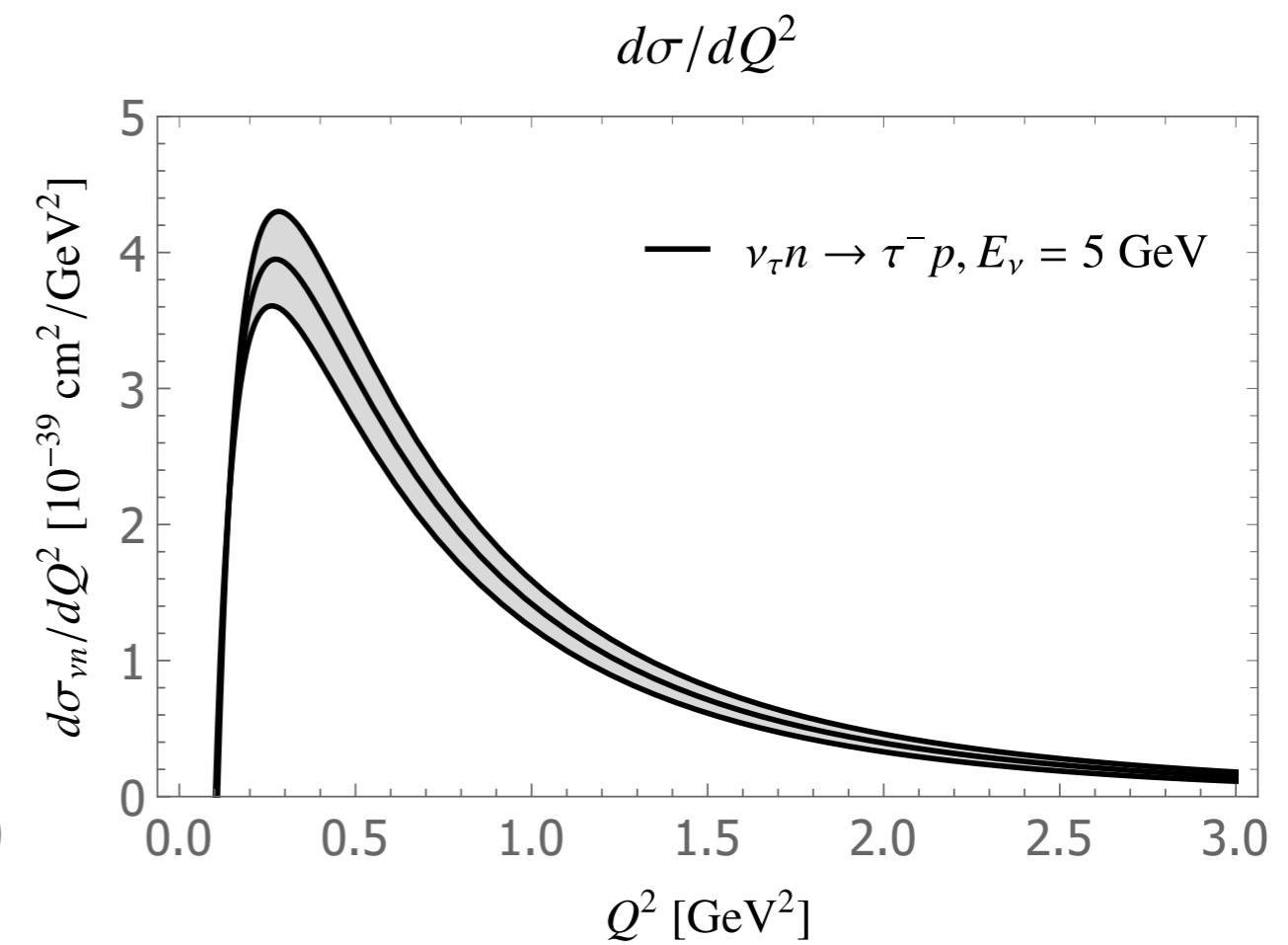
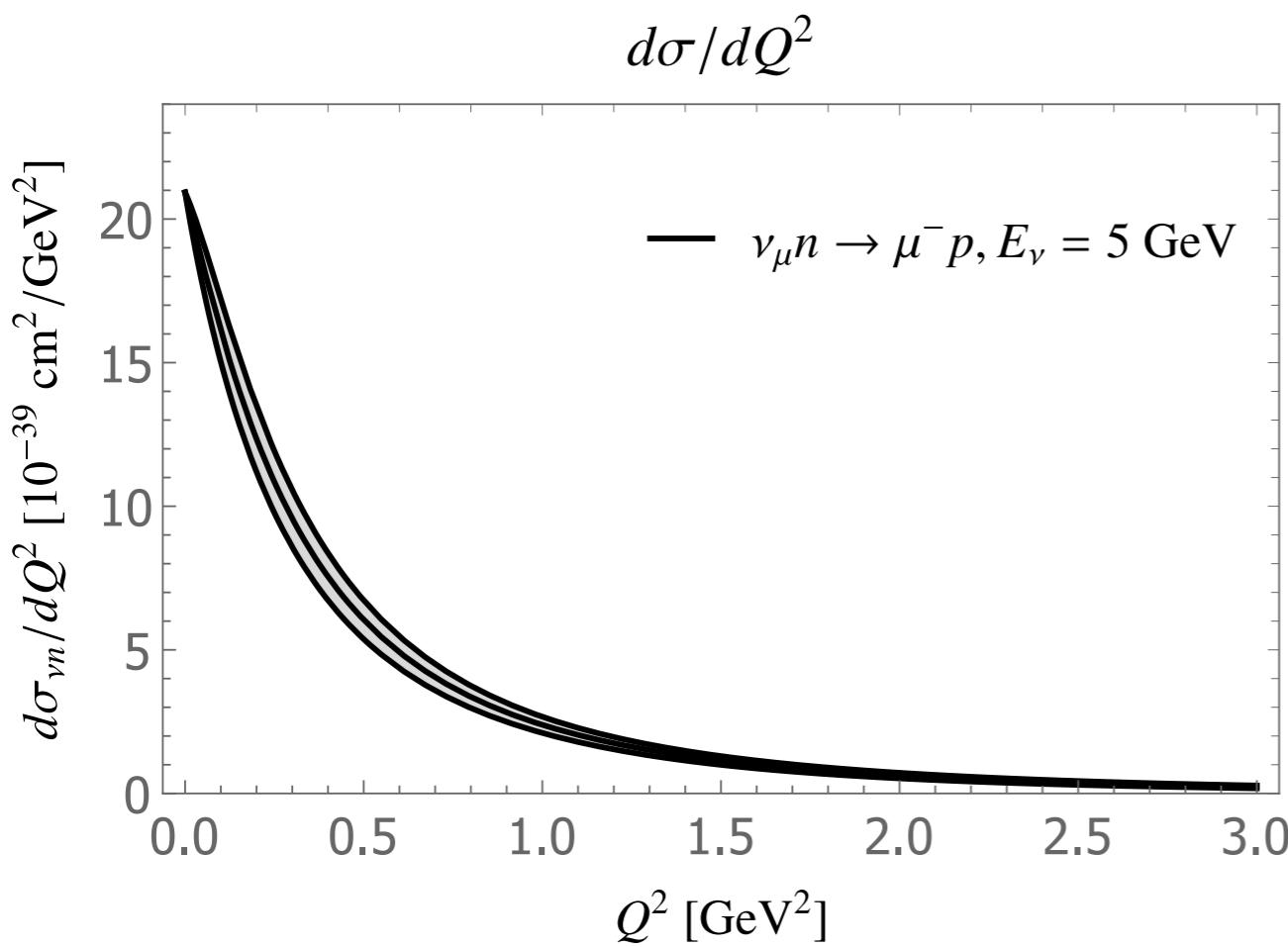
$$A = 2\tau(F_D^V + F_P^V)^2 - (1 + \tau) \left[(F_D^V)^2 + \tau(F_P^V)^2 - (F_A)^2 \right]$$
$$- \frac{m_\ell^2}{4M^2} \left[(F_D^V + F_P^V)^2 + (F_A + 2F_P)^2 - 4(1 + \tau)F_P^2 \right]$$

$$B = 4\tau F_A (F_D^V + F_P^V) \quad C = \frac{1}{4} \left[(F_D^V)^2 + \tau(F_P^V)^2 + (F_A)^2 \right]$$

- cross section is sensitive to both vector and axial contributions

CCQE scattering cross section

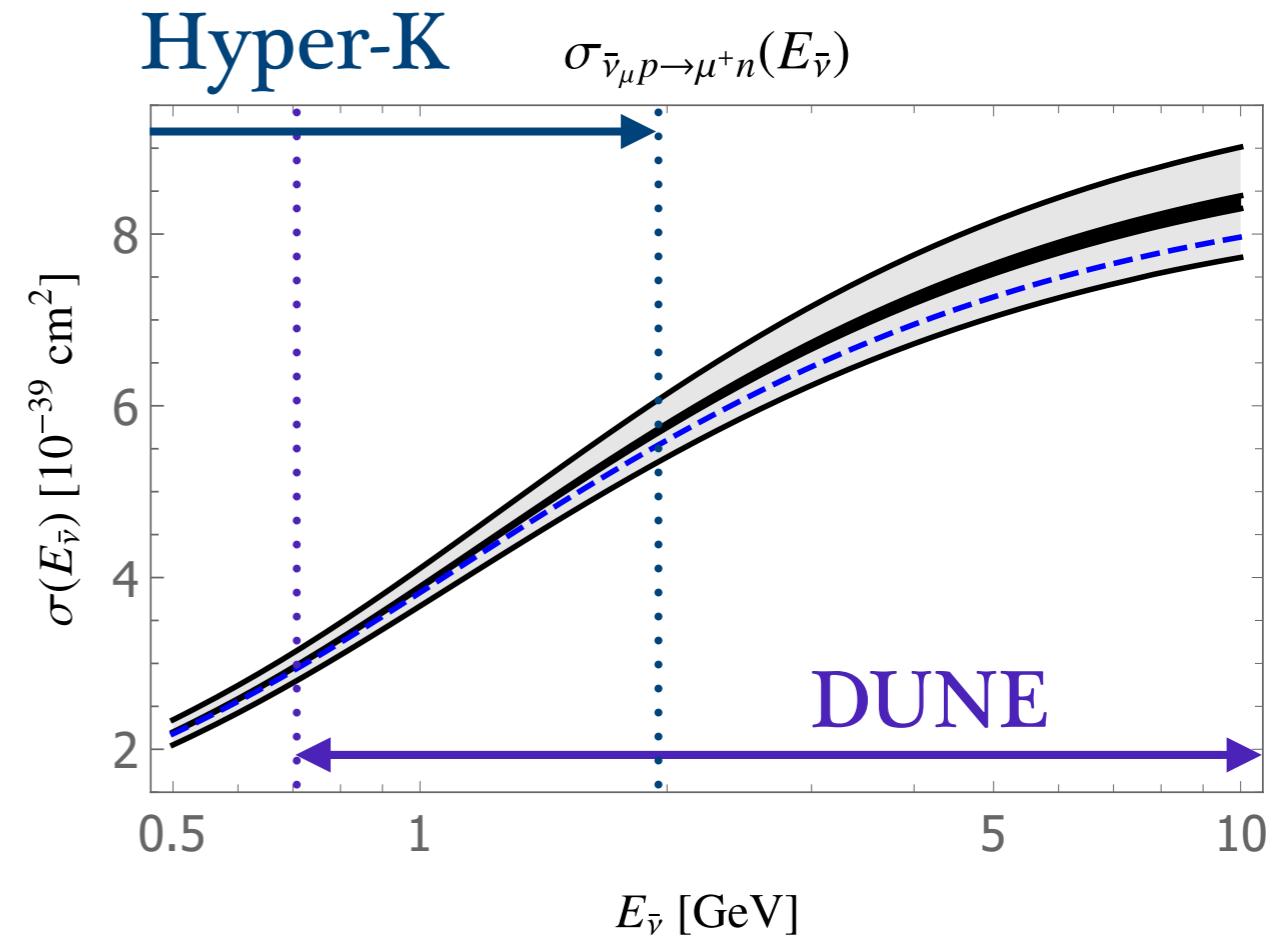
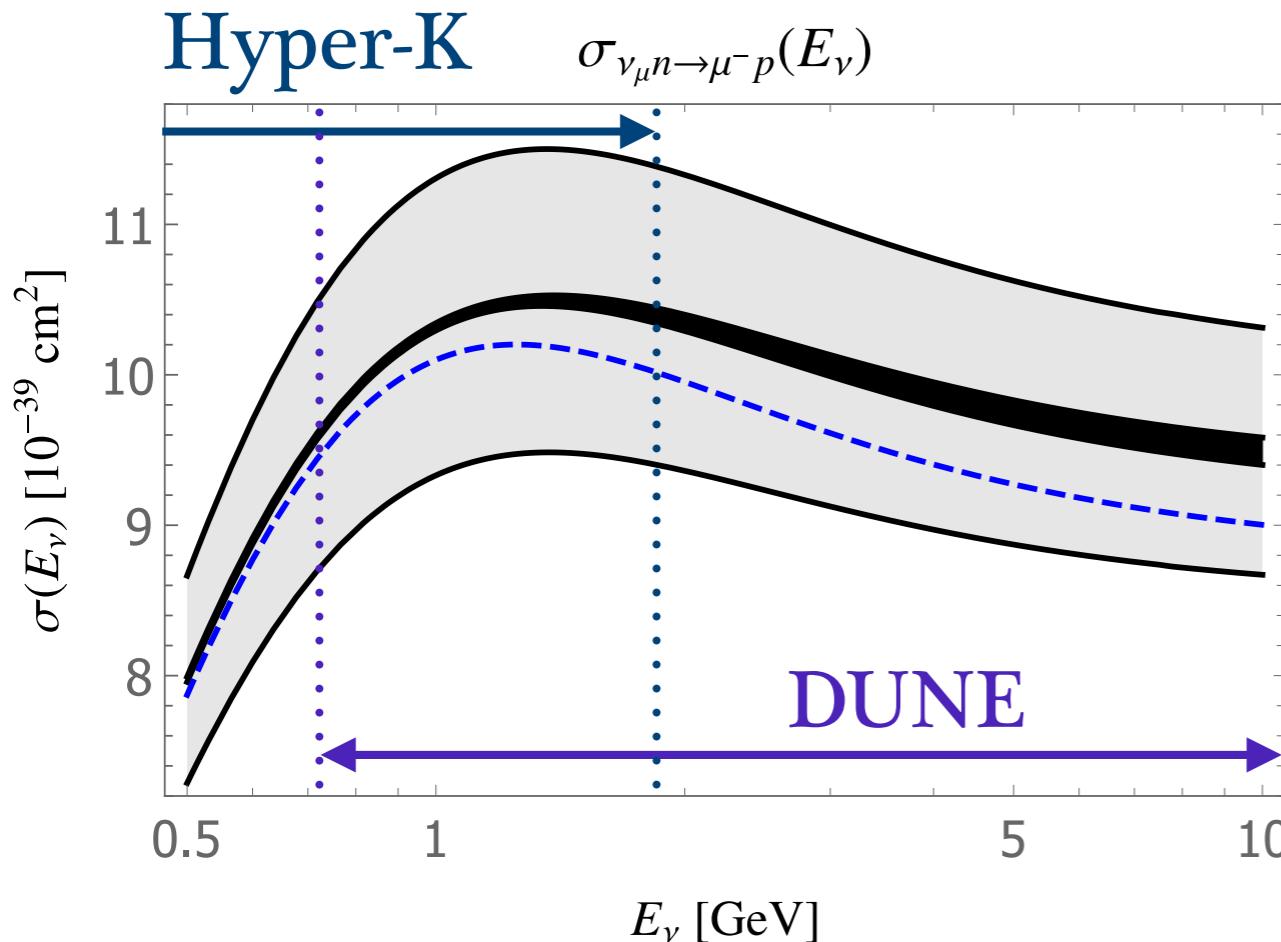
- cross section is energy-independent above a few GeV neutrino energy
- relevant kinematic range of structure effects:



- cross section is dominated by hadronic scales

CCQE scattering cross section

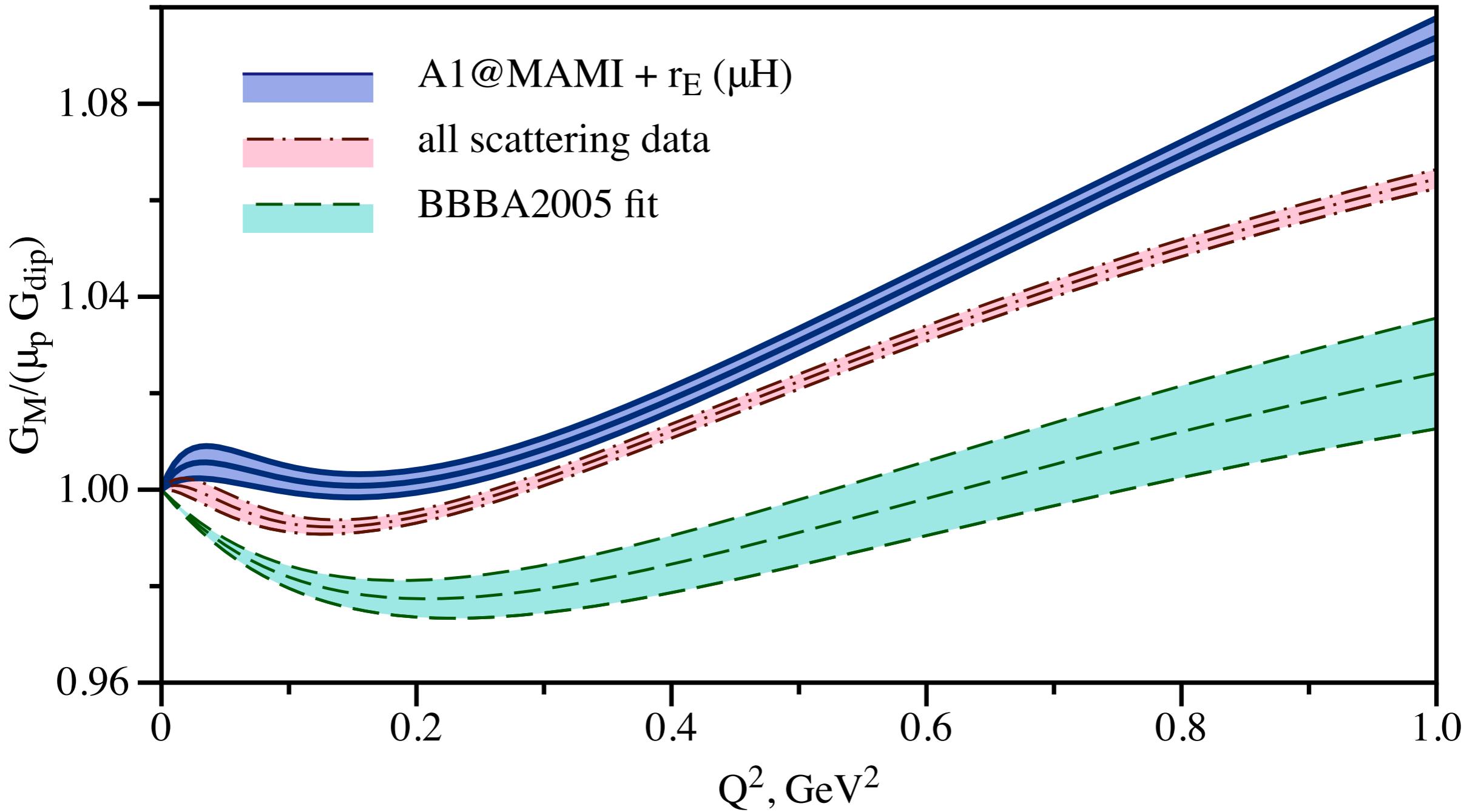
- dark band: uncertainty of iso 1 fit
- light band: uncertainty of axial form factor
- blue line: BBBA2005 fit of electromagnetic form factors



- knowledge of vector structure stops a progress in studies of axial

Origin of difference

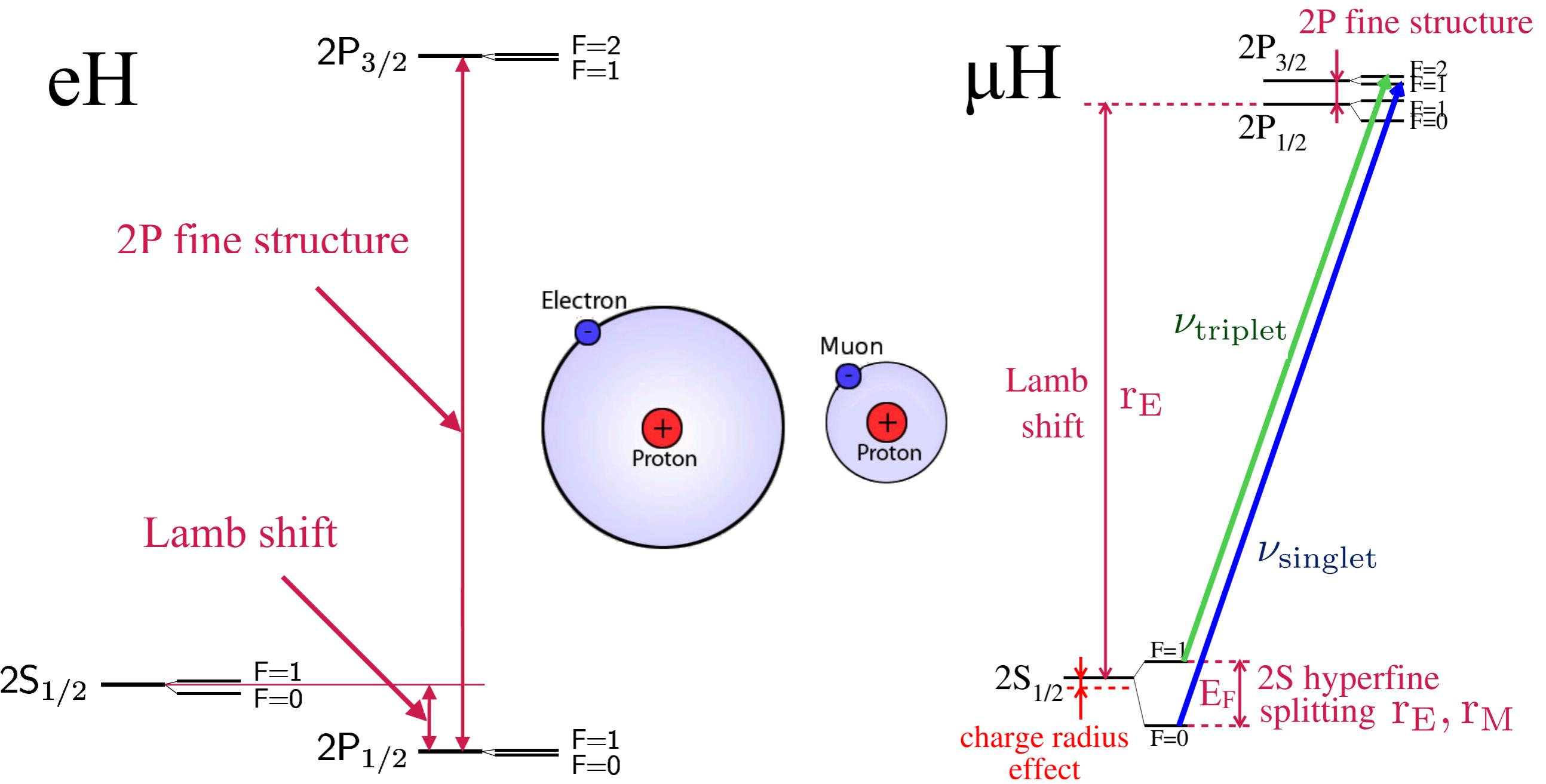
- fits of proton magnetic form factor:



- proton magnetic form factor has to be precisely measured again

1S hyperfine splitting in muonic hydrogen

Lamb shift and hyperfine splitting in H



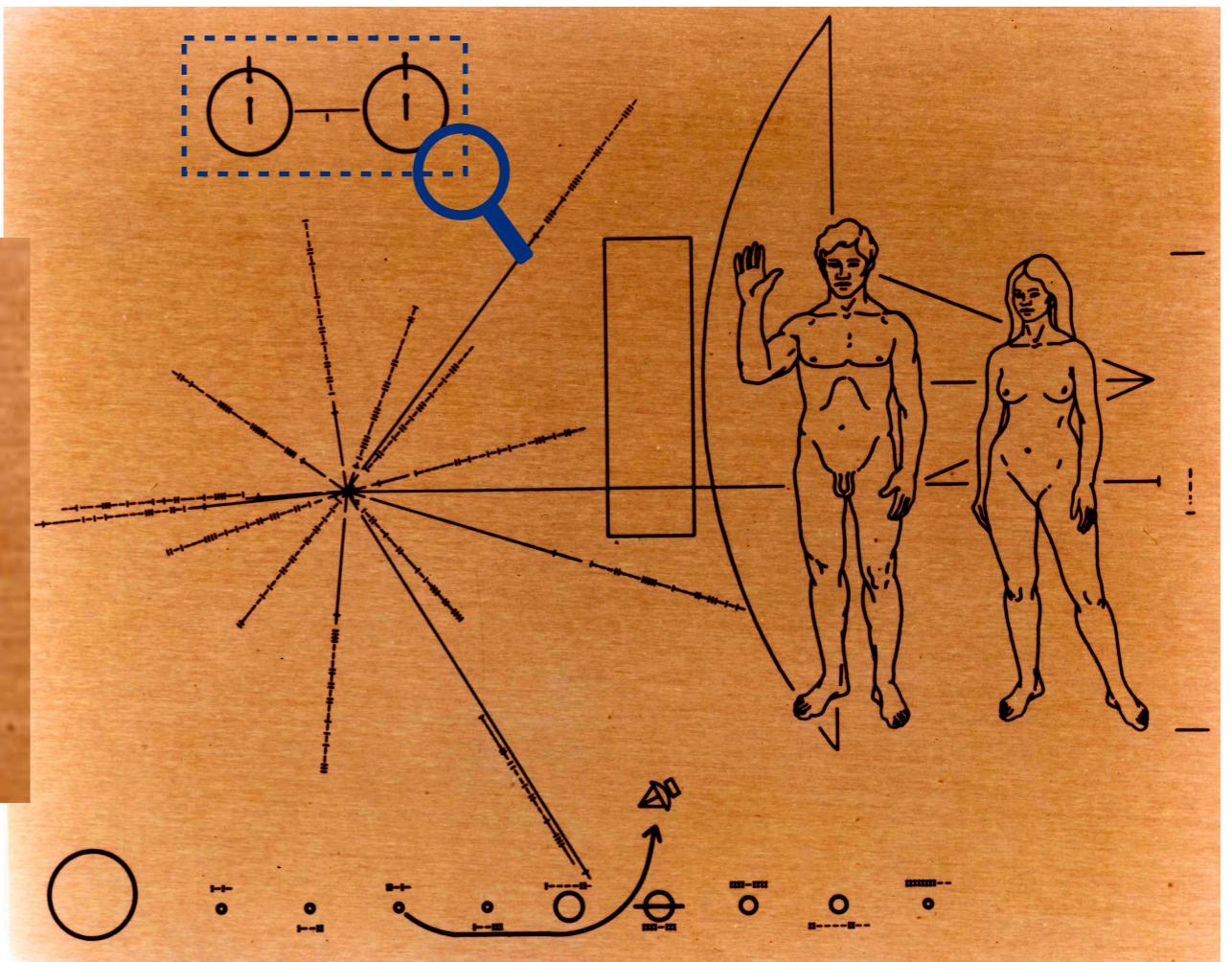
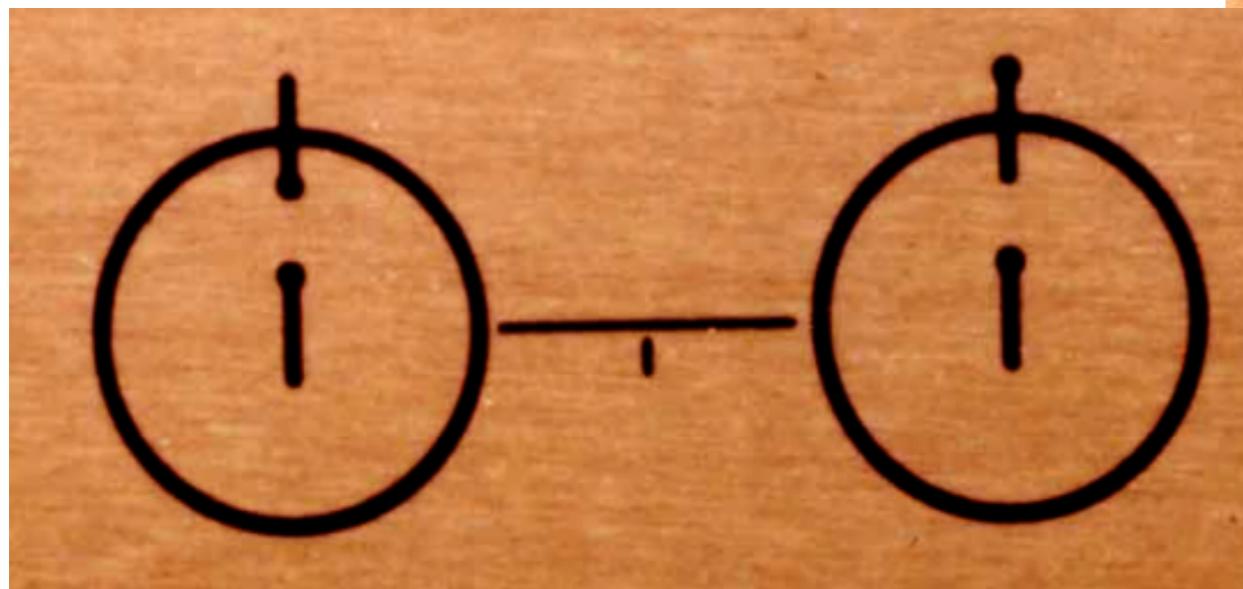
- 1S HFS in μH with 1 ppm accuracy at PSI, J-PARC, RIKEN-RAL

eH 1S HFS

- measurements of 1S HFS in eH (21 cm line):

$$\nu_{\text{HFS}}(\text{H}) = 1420.4057517667(9) \text{ MHz}$$

1970th



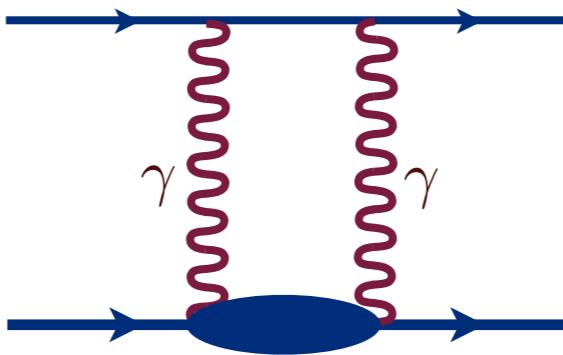
Pioneer plaque, NASA Ames

- relation between eH and μH through 2γ

Pineda and Peset (2017), O. T. (2018, 2019)

Hyperfine splitting and Zemach radius

- 2γ provides the leading proton-structure correction to HFS



- experimental collaborations aim to extract Zemach radius

$$r_Z = -\frac{4}{\pi} \int_0^\infty \frac{dQ}{Q^2} \frac{G_M(Q^2) G_E(Q^2) - G_M(0) G_E(0)}{G_M(0)}$$

dominant piece of 2γ with proton intermediate state

$$\Delta E_Z = -2\alpha m_r E_F r_Z$$

	proton (fm)	neutron (fm)
p, n	1.0227(94)(51)	-0.0443(26)(1)
iso 1	1.0246(84)(40)	-0.0445(14)(3)
iso 3	1.0450(58)(45)	-0.0462(12)(0)
previous estimates	1.045-1.086	-0.0449(13)

- Zemach radius is sensitive to radii and proton magnetic form factor

1S-2S transition in hydrogen

1S-2S transition in hydrogen and 2γ

- measurements of 1S-2S transition in eH with 4×10^{-15} accuracy:

$$\nu_{1S-2S}(H) = 2466061413187018(11) \text{ Hz} \quad \text{2010th}$$

A. Matveev et al. (2013)

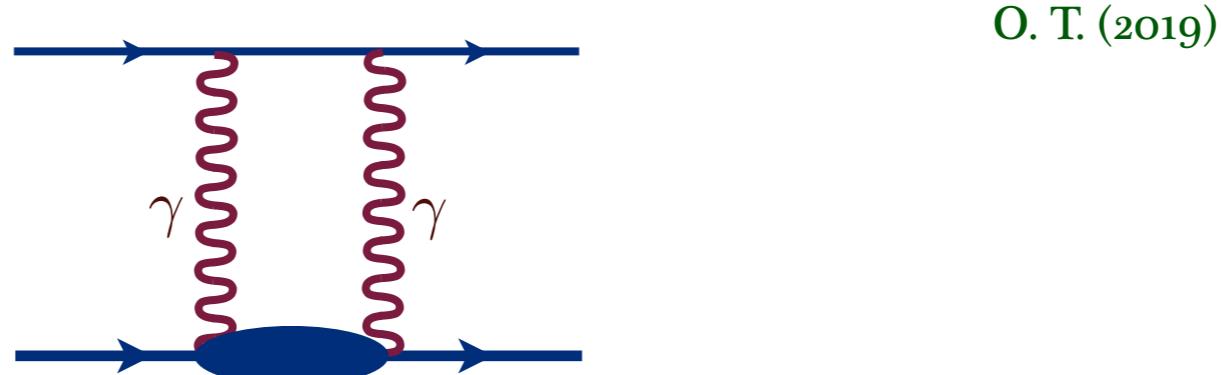
- more precise than recent Lamb shift measurement (error: 3.2 kHz)

N. Bezginov et al. (2019)

$$\nu_{nS} = -\frac{R_\infty}{n^2} + \frac{L_{1S}(r_E)}{n^3}$$

- main input to determine Rydberg constant

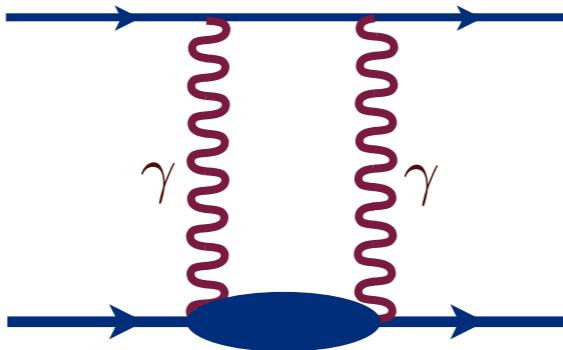
- 2γ uncertainty is at the level of experimental precision



O. T. (2019)

1S-2S transition in hydrogen and 2γ

- 2γ provides a sizeable theoretical uncertainty to S energy levels



important also for extraction of charge radius from μ H Lamb shift

- bulk of correction is given by the Friar radius (p intermediate state)

$$r_F^{(3)} = \frac{48}{\pi} \int_0^{\infty} \frac{dQ}{Q^2} \frac{G_E^2(Q^2) - G_E^2(0) - 2Q^2 G'_E(0)}{Q^2}$$

	proton (fm ³)	neutron (fm ³)
p, n	2.246(58)(2)	0.0093(11)(1)
iso 1	2.278(49)(12)	0.0093(6)(1)
iso 3	2.176(38)(10)	0.0100(5)(1)
previous estimates	2.71-2.89	small

- Friar radius is sensitive to charge form factor and radius

Conclusions

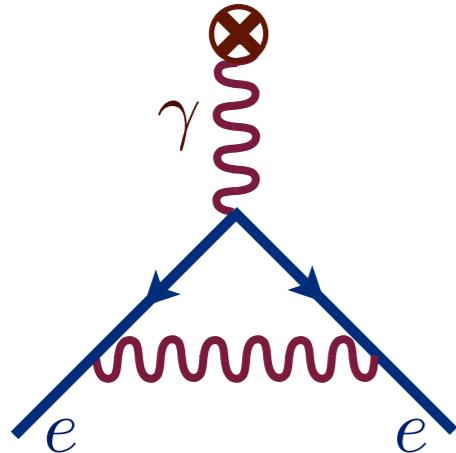
- nucleon electromagnetic form factors are a crucial input for precise probes with nucleons and nuclei
- form factor fit from relevant kinematical region is presented in a convenient form for applications, i.e., neutrino event generators like GENIE
- including data of A1@MAMI Collaboration, CCQE cross sections shift by 3-5 % triggered by proton magnetic form factor
- future experimental and lattice QCD studies of all form factors will be very helpful for neutrino and atomic physics



Thanks for your attention !!!

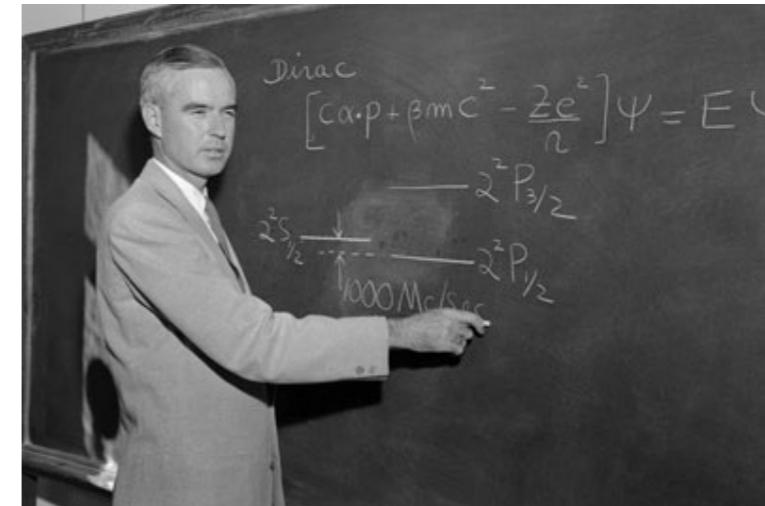
Muon discrepancies: new physics?

anomalous magnetic moment



Polykarp Kusch (1947)

Lamb shift

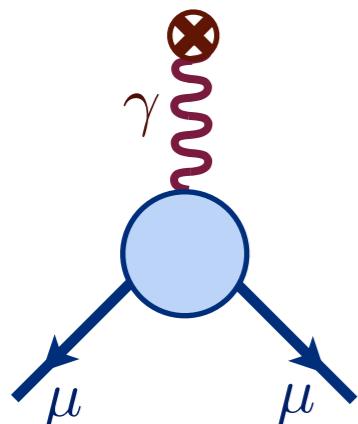


W. Lamb and R. Rutherford (1947)

formulation of QED as first successful QFT

J. Schwinger, F. Dyson, R. Feynman, S. Tomonaga (1947-1949)

anomalous magnetic moment



3.6 σ
theory vs exp.



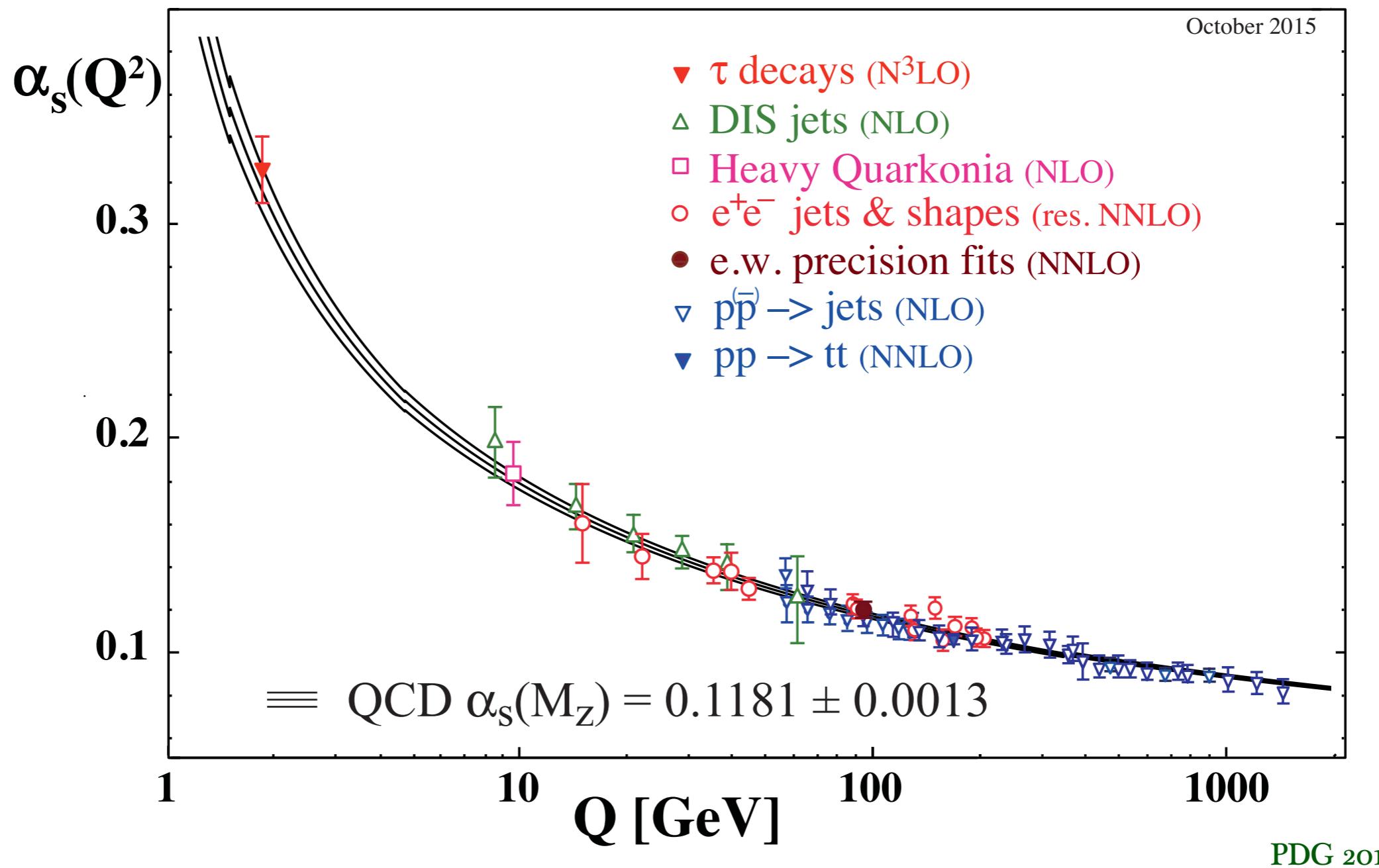
μ H Lamb shift

proton size
discrepancy
initially 5-6 σ
 e H, ep vs μ H

- hadronic uncertainty is dominant in theory

QCD running coupling

- asymptotic freedom at high energies



- EFTs, lattice QCD and phenomenology at low energies

Future projects

- ep elastic scattering at low Q^2 :

ProRad@PRAE

$\omega = 30 - 70$ MeV

$Q^2 = 10^{-5} - 10^{-4}$ GeV 2

ep scattering@MAMI

$\omega = 500$ and 720 MeV

$Q^2 = 0.002 - 0.02$ GeV 2

Ultra-Low Q^2 @Tohoku

$\omega = 20 - 60$ MeV

$Q^2 = 0.0003 - 0.008$ GeV 2

ISR@MAMI

$\omega = 0.2 - 0.5$ GeV

$Q^2 = 0.001 - 0.17$ GeV $^2 \rightarrow 0.0002$ GeV 2

MAGIX@MESA

magnetic form factor

Future projects

- μp elastic scattering at low Q^2 :

MUSE@PSI

$\omega = 115 - 210 \text{ MeV}$

$Q^2 = 0.0016 - 0.08 \text{ GeV}^2$

e and μ

COMPASS@CERN

$\omega = 100 \text{ GeV}$

$Q^2 = 0.001 - 0.2 \text{ GeV}^2$

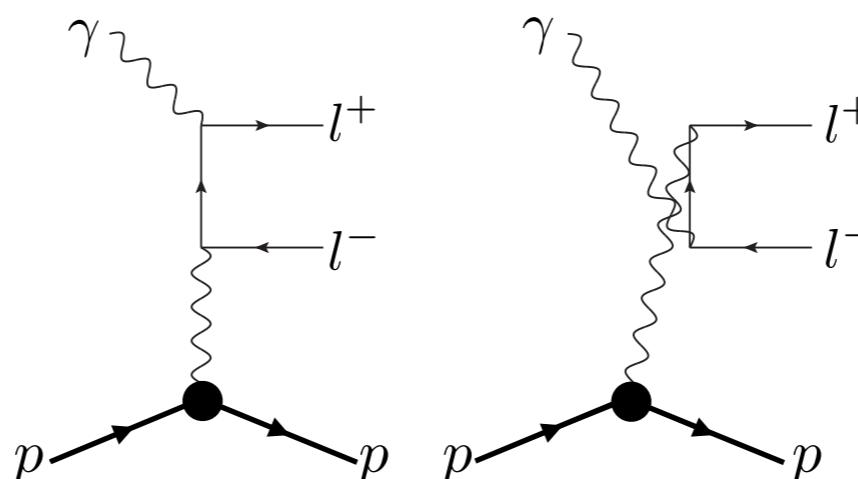
small structure effects

- universality test by lepton-pair photoproduction:

$\gamma p \rightarrow l^+ l^- p$ @MAMI

$\omega = 0.5 - 1.5 \text{ GeV}$

$Q^2 = 0.0018 - 0.042 \text{ GeV}^2$



$$\frac{\sigma(e^+e^-) + \sigma(\mu^+\mu^-)}{\sigma(e^+e^-)}$$

below and above
muon threshold

normalisation and proton structure errors are suppressed

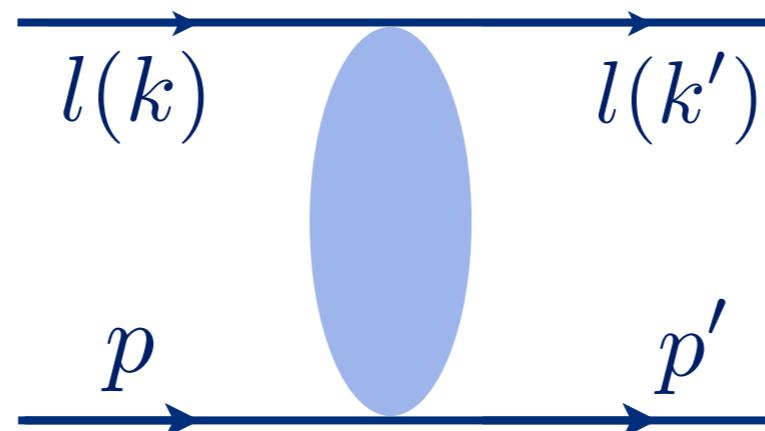
Elastic lepton-proton scattering and 2γ

momentum transfer

$$Q^2 = -(k - k')^2$$

crossing-symmetric
variable

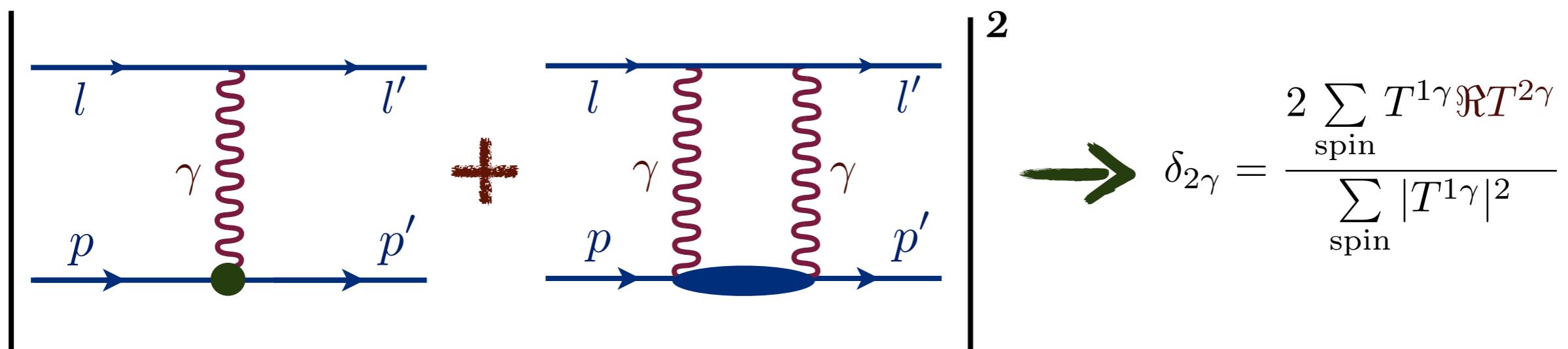
$$\nu = \frac{(k, p + p')}{2}$$



photon polarization
parameter
 ε

forward scattering
 $\varepsilon \rightarrow 1$

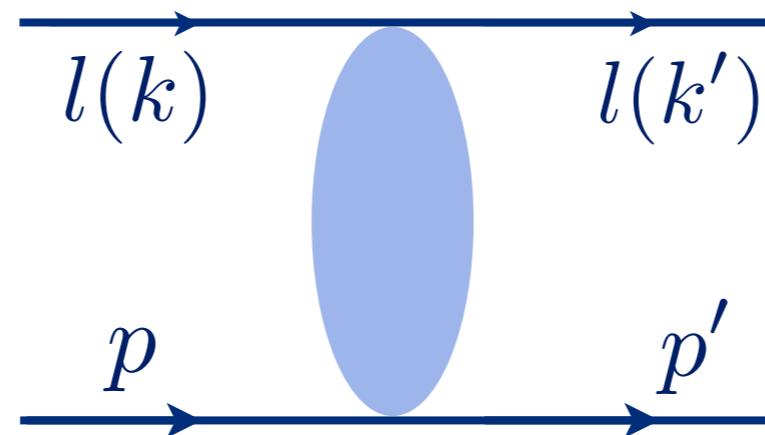
- leading 2γ contribution: interference term



- 2γ correction to cross section is given by amplitudes real parts

Elastic lepton-proton scattering and 2γ

$$K = \frac{k + k'}{2}$$



$$P = \frac{p + p'}{2}$$

- electron-proton scattering: 3 structure amplitudes

$$T^{\text{non-flip}} = \frac{e^2}{Q^2} \bar{l} \gamma_\mu l \cdot \bar{N} \left(\mathcal{G}_M(\nu, Q^2) \gamma^\mu - \mathcal{F}_2(\nu, Q^2) \frac{P^\mu}{M} + \mathcal{F}_3(\nu, Q^2) \frac{\hat{K} P^\mu}{M^2} \right) N$$

P.A.M. Guichon and M. Vanderhaeghen (2003)

- real parts of all 2γ amplitudes extracted at one point $Q^2=2.5$ GeV²

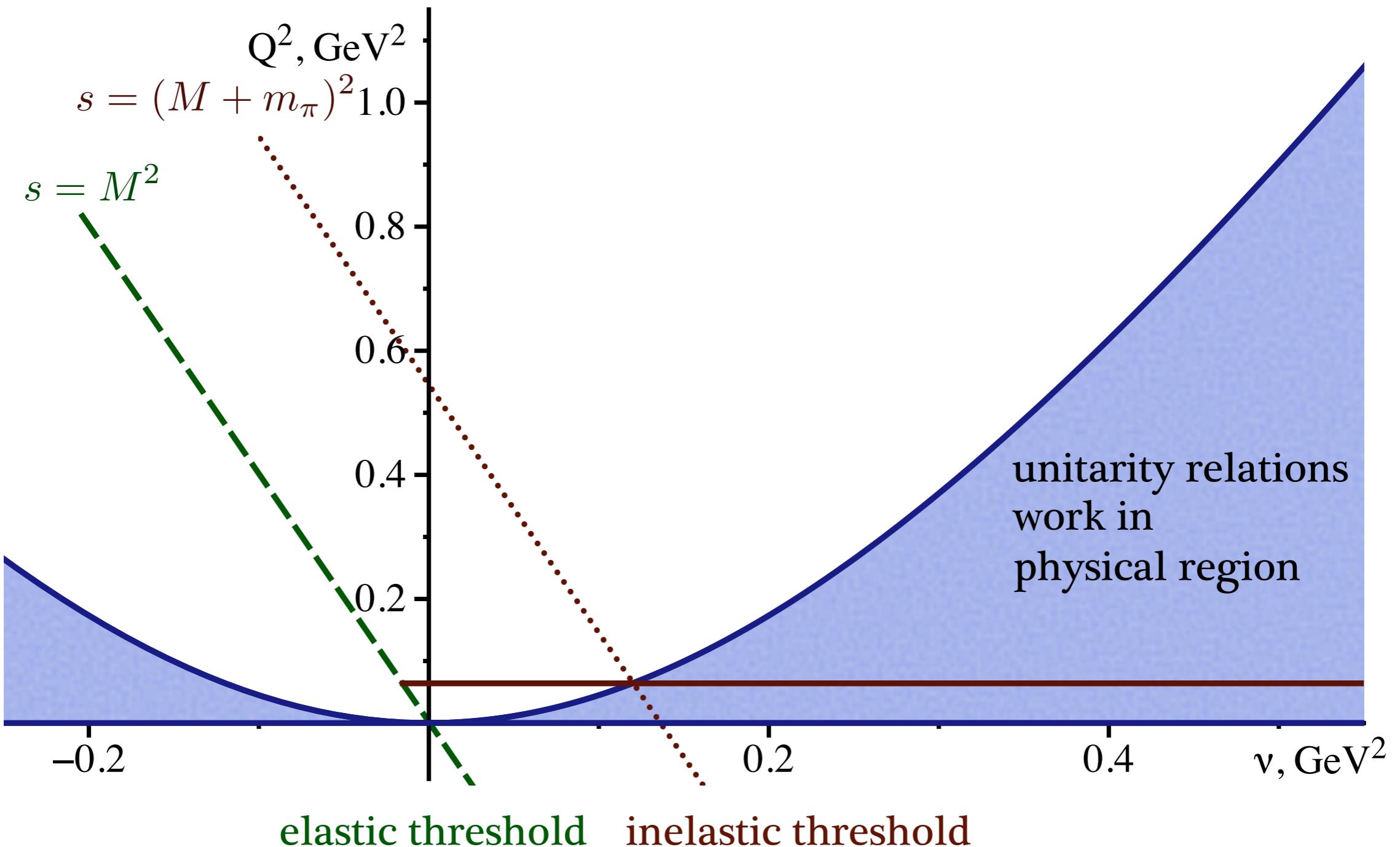
J. Guttmann, N. Kivel, M. Meziane and M. Vanderhaeghen (2011)

Dm. Borisyuk and A. Kobushkin (2011)

I. A. Qattan (2017-2018)

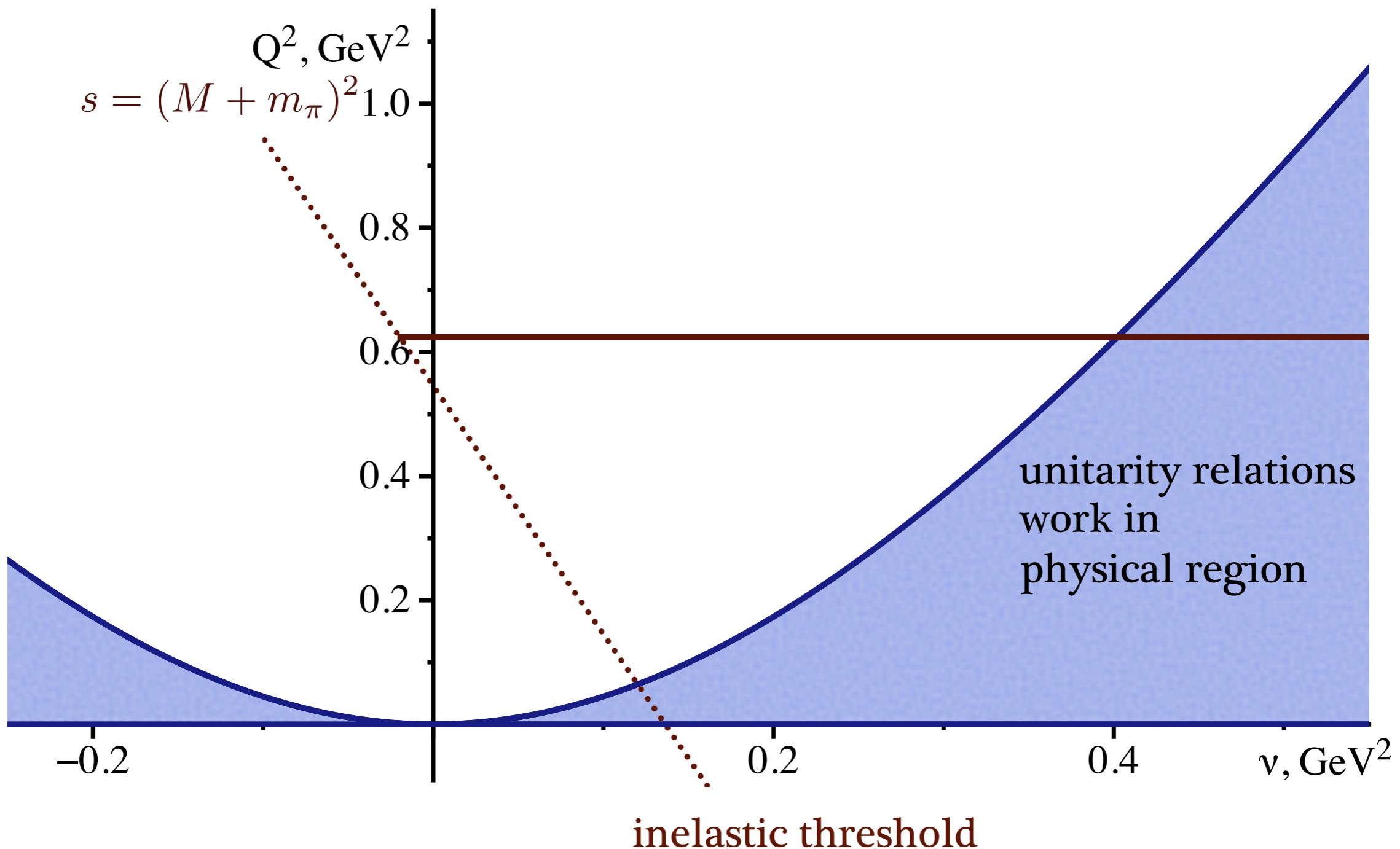
- 2γ correction to cross section is given by amplitudes real parts

Mandelstam plot (ep)



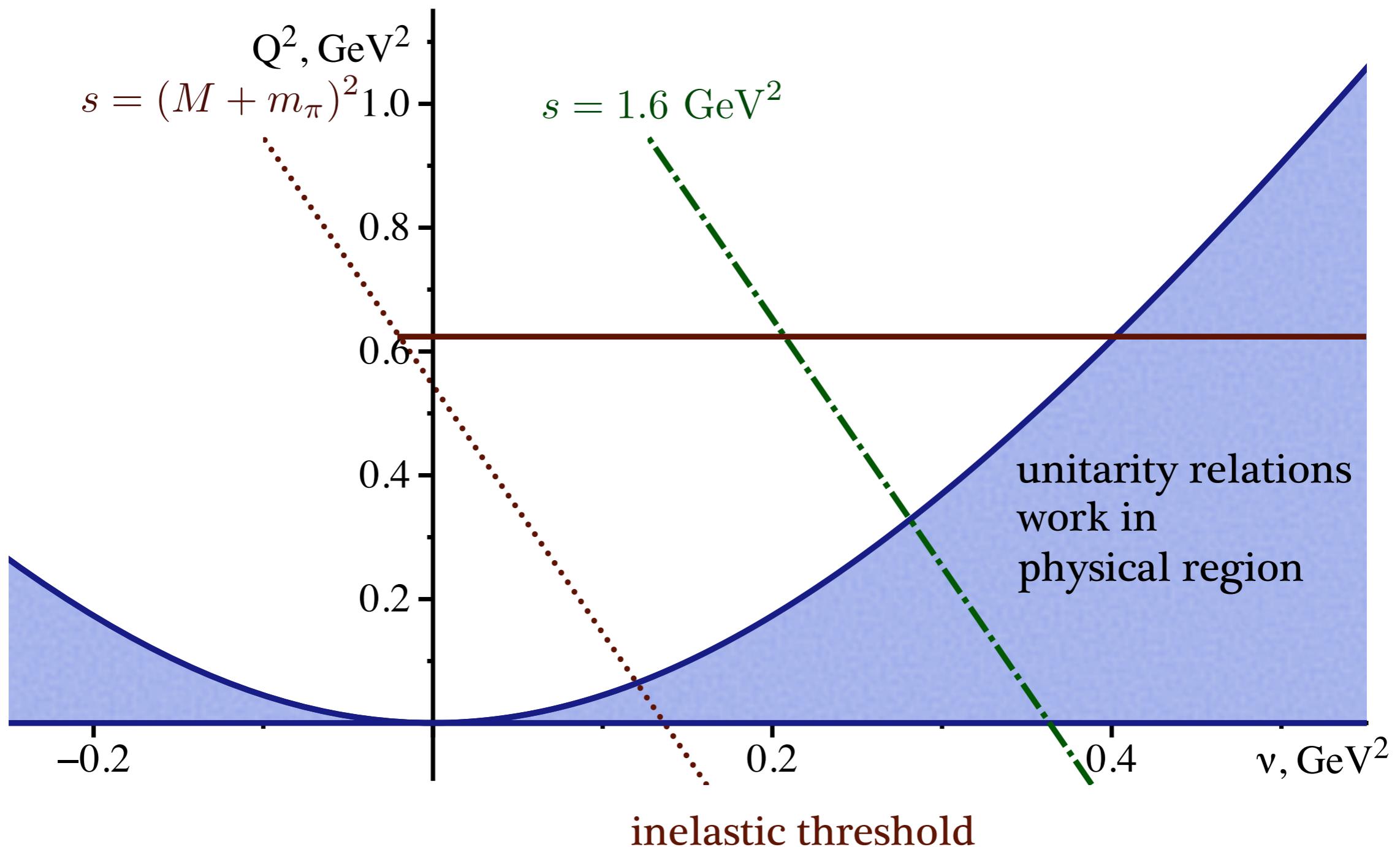
- proton intermediate state is **outside** physical region for $Q^2 > 0$
- πN intermediate state is **outside** physical region for $Q^2 > 0.064 \text{ GeV}^2$

Mandelstam plot (ep)



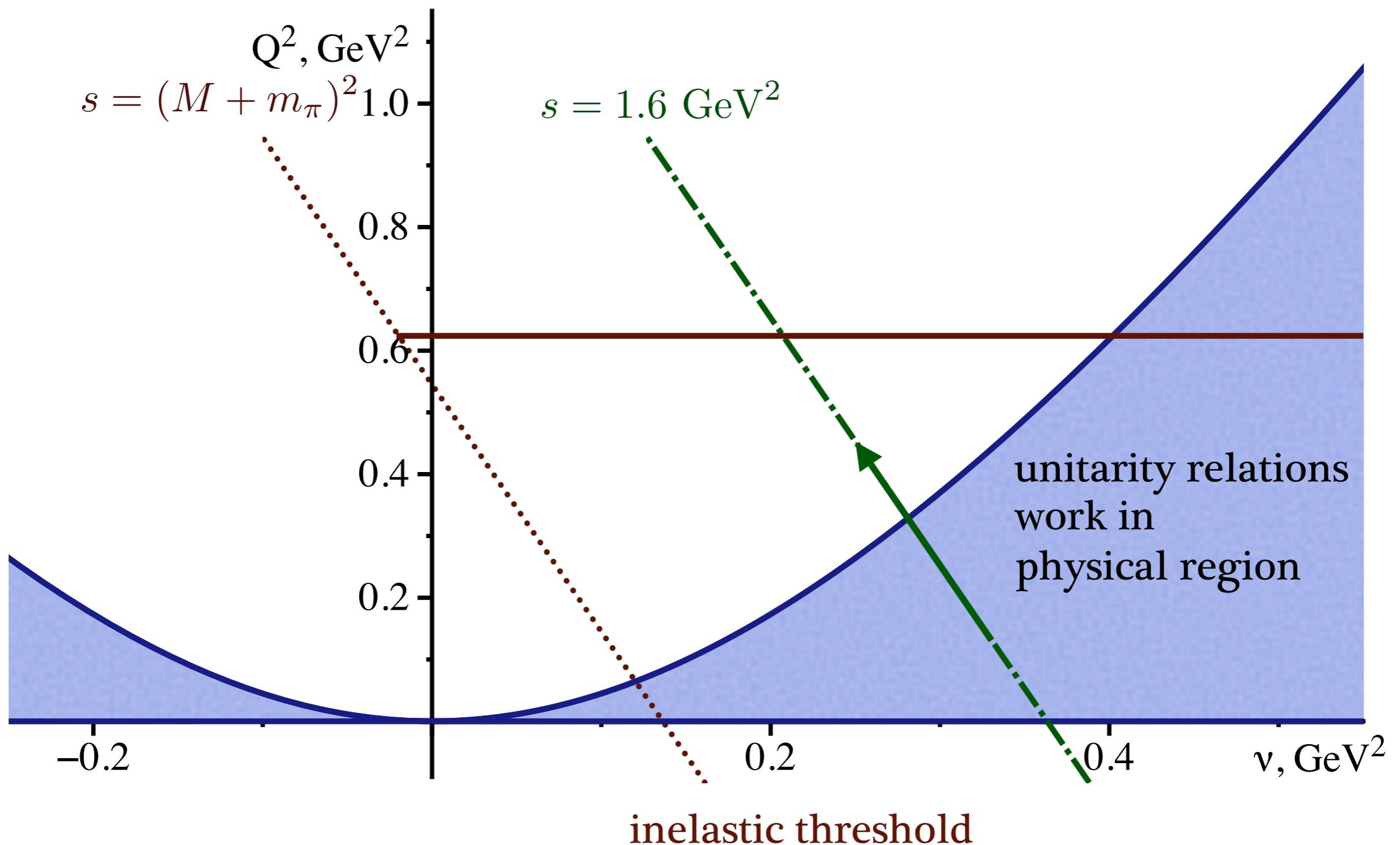
- πN intermediate state is **outside** physical region for $Q^2 > 0.064 \text{ GeV}^2$

Mandelstam plot (ep)



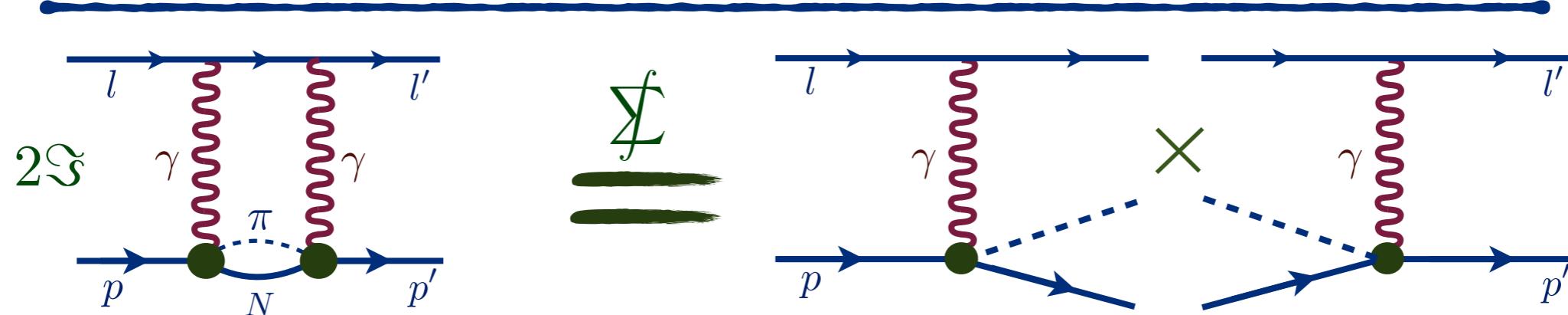
- πN intermediate state is **outside** physical region for $Q^2 > 0.064 \text{ GeV}^2$

Mandelstam plot (ep)



- πN intermediate state is **outside** physical region for $Q^2 > 0.064 \text{ GeV}^2$

Analytical continuation. πN states

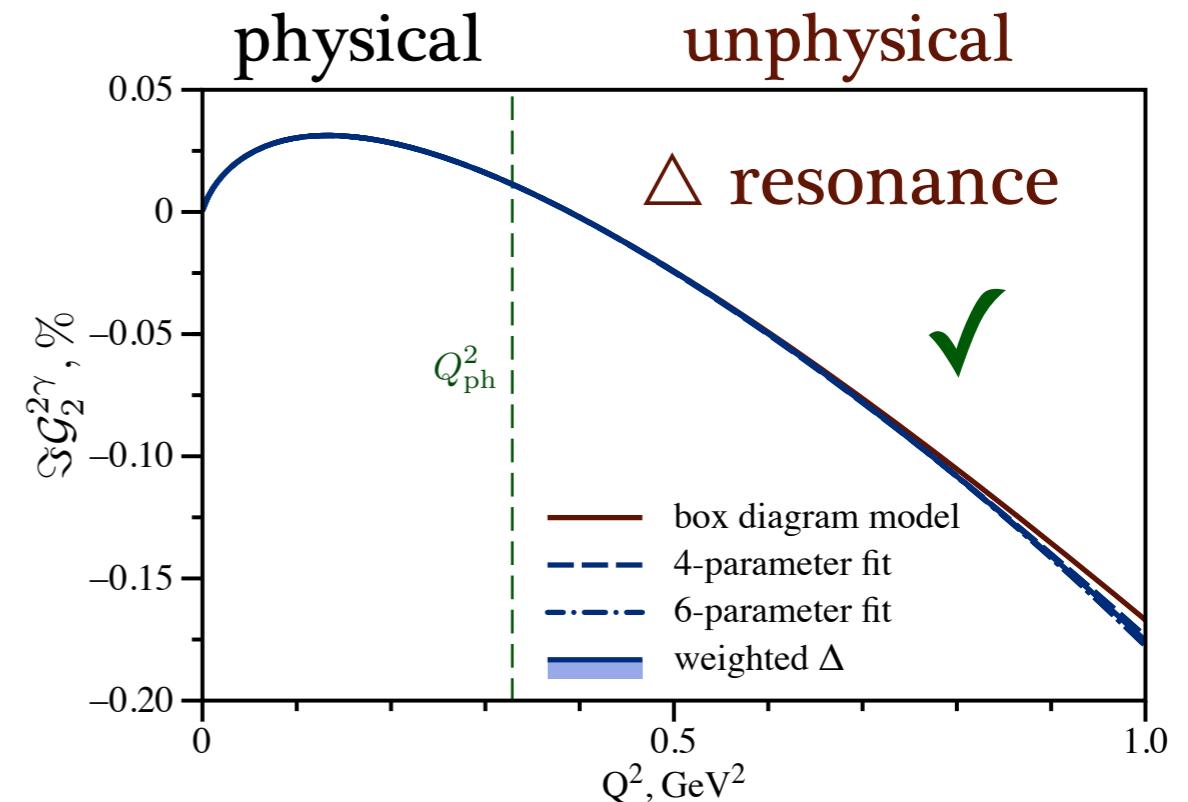
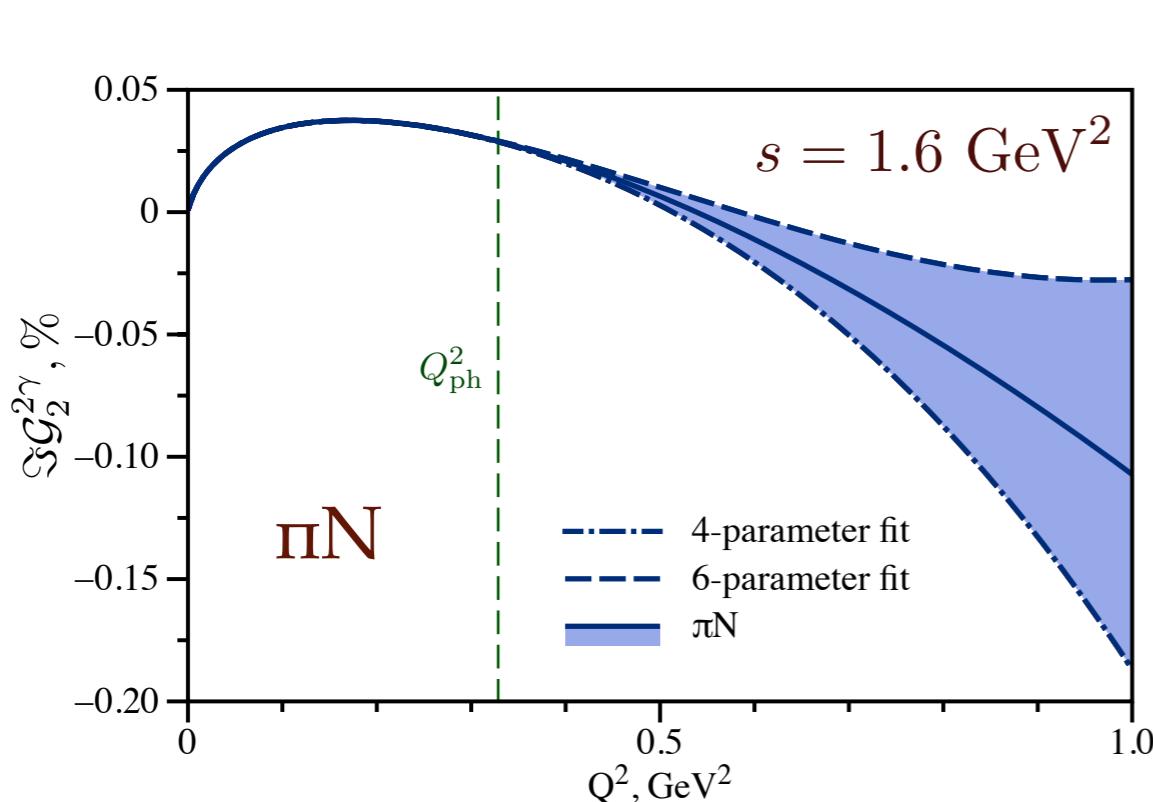


- pion electroproduction amplitudes: **MAID2007**

D. Drechsel, S. Kamalov and L. Tiator (2007)

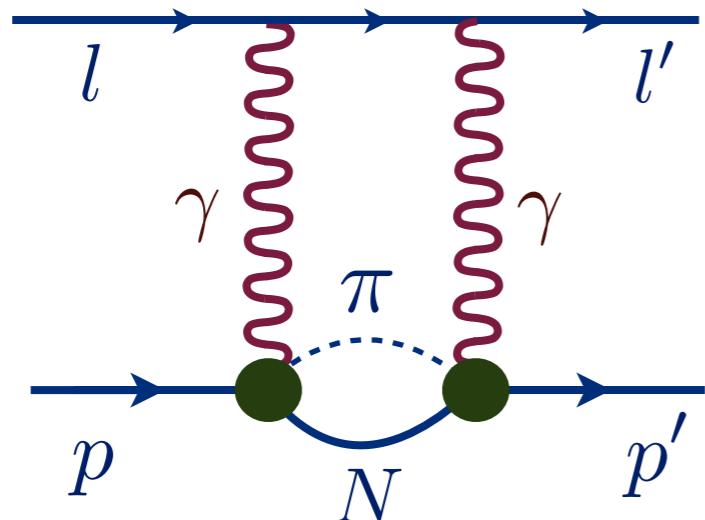
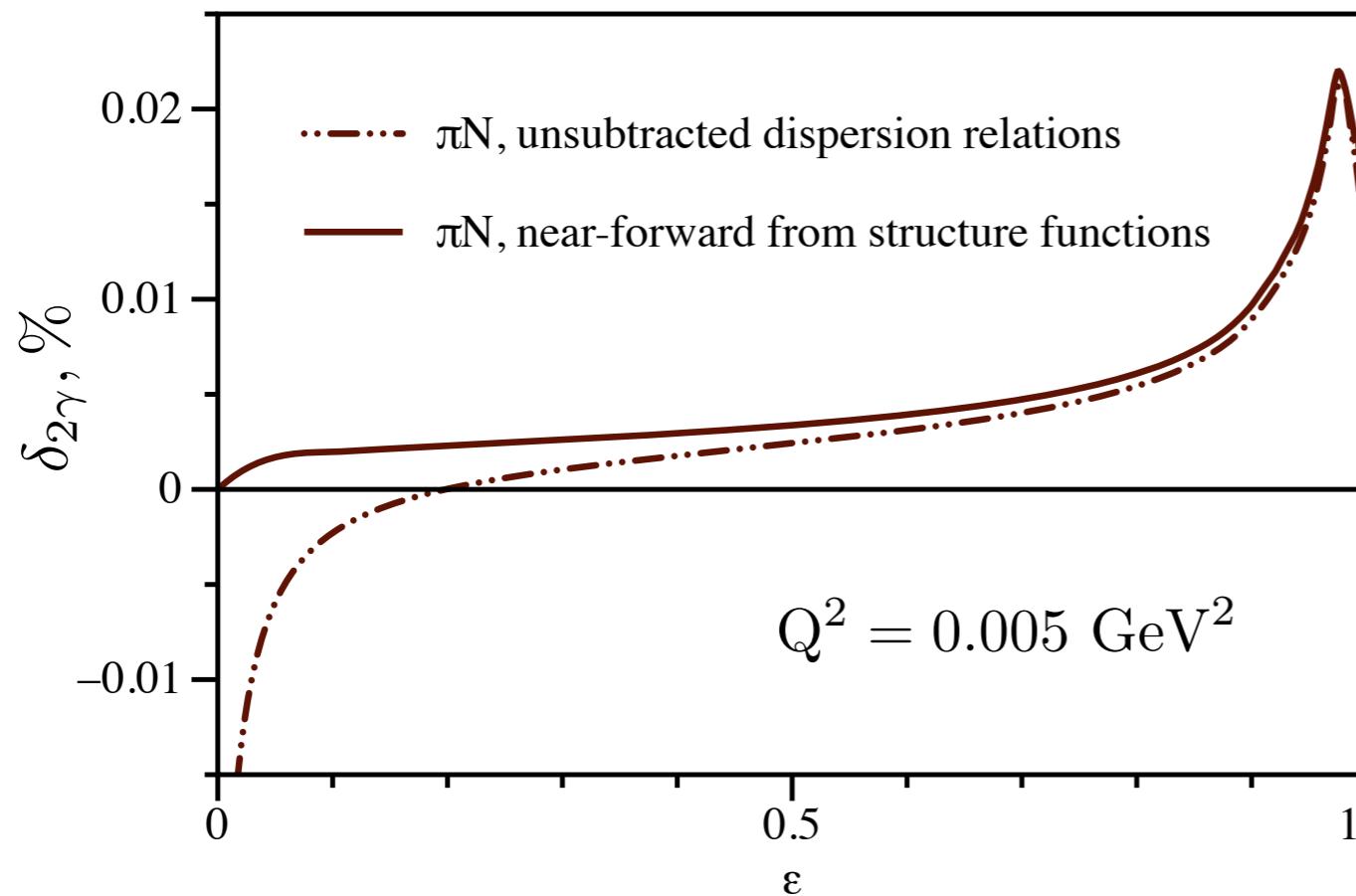
- analytical continuation: fit of **low- Q^2 expansion** in physical region

$$\mathcal{G}_{1,2}(s, Q^2), Q^2 \mathcal{F}_3(s, Q^2) \sim a_1 Q^2 \ln Q^2 + a_2 Q^2 + a_3 Q^4 \ln Q^2 + \dots$$

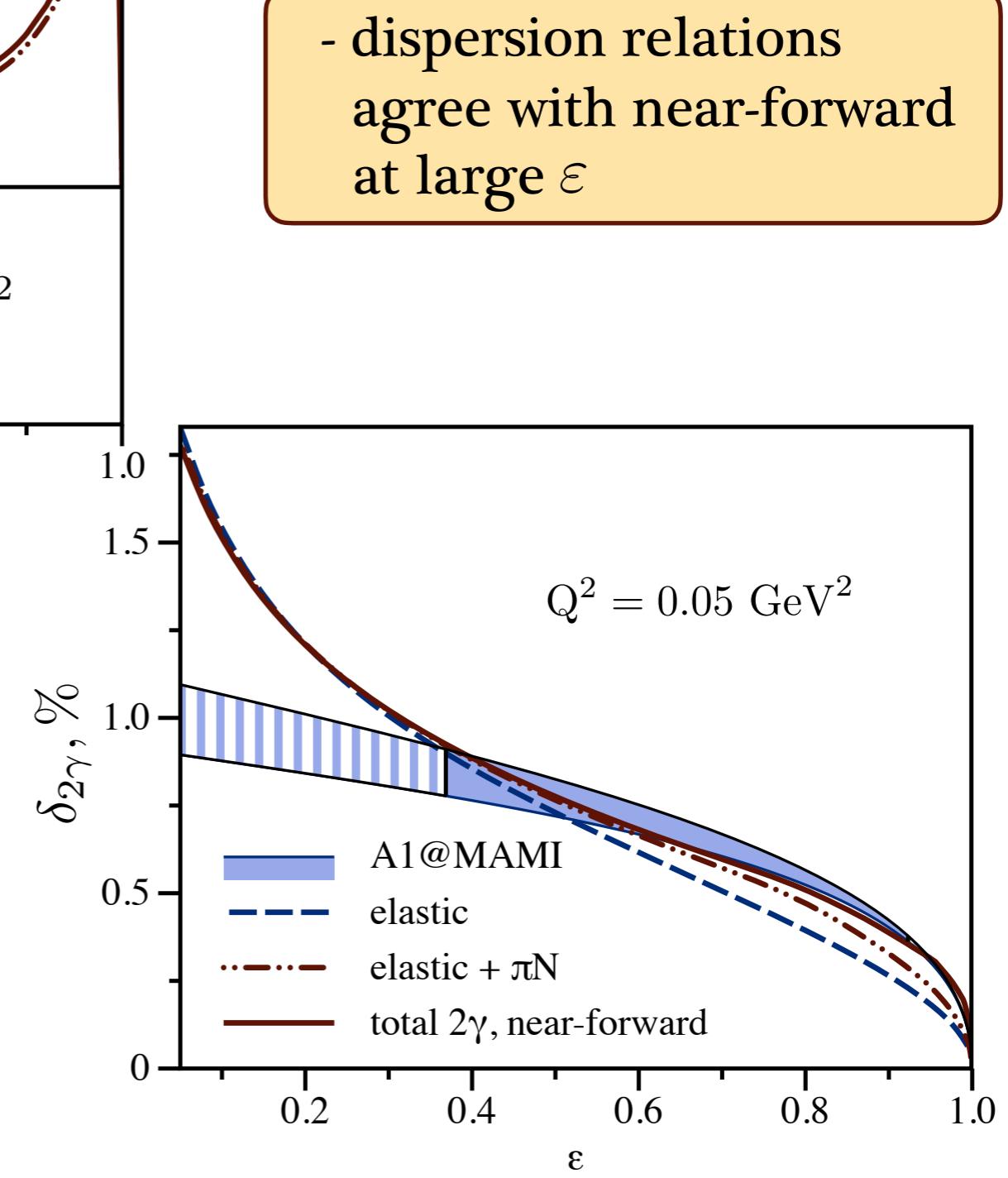


- uncertainty: extrapolation + large invariant masses

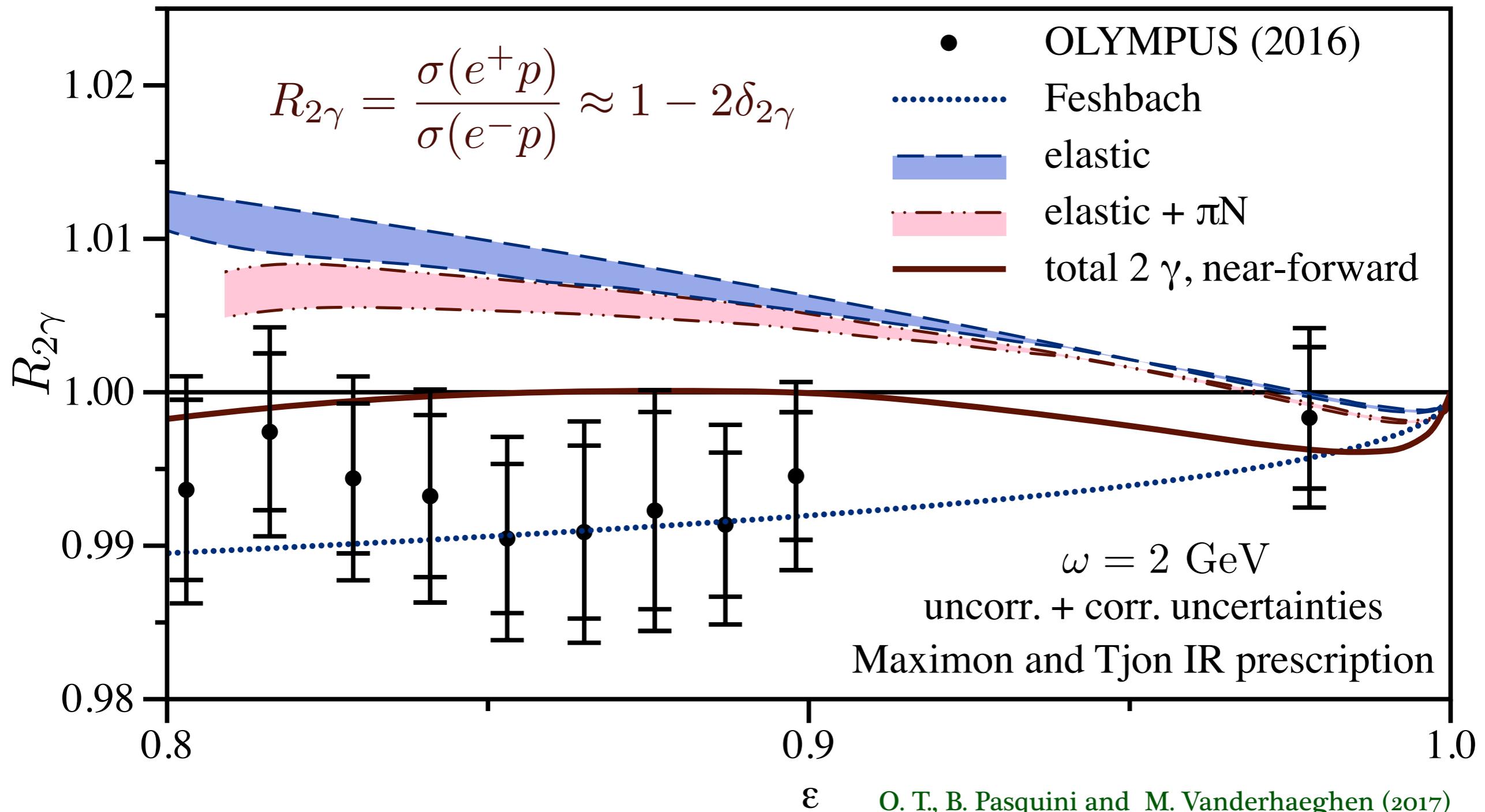
πN in dispersive framework (e-p)



- πN is dominant inelastic 2γ

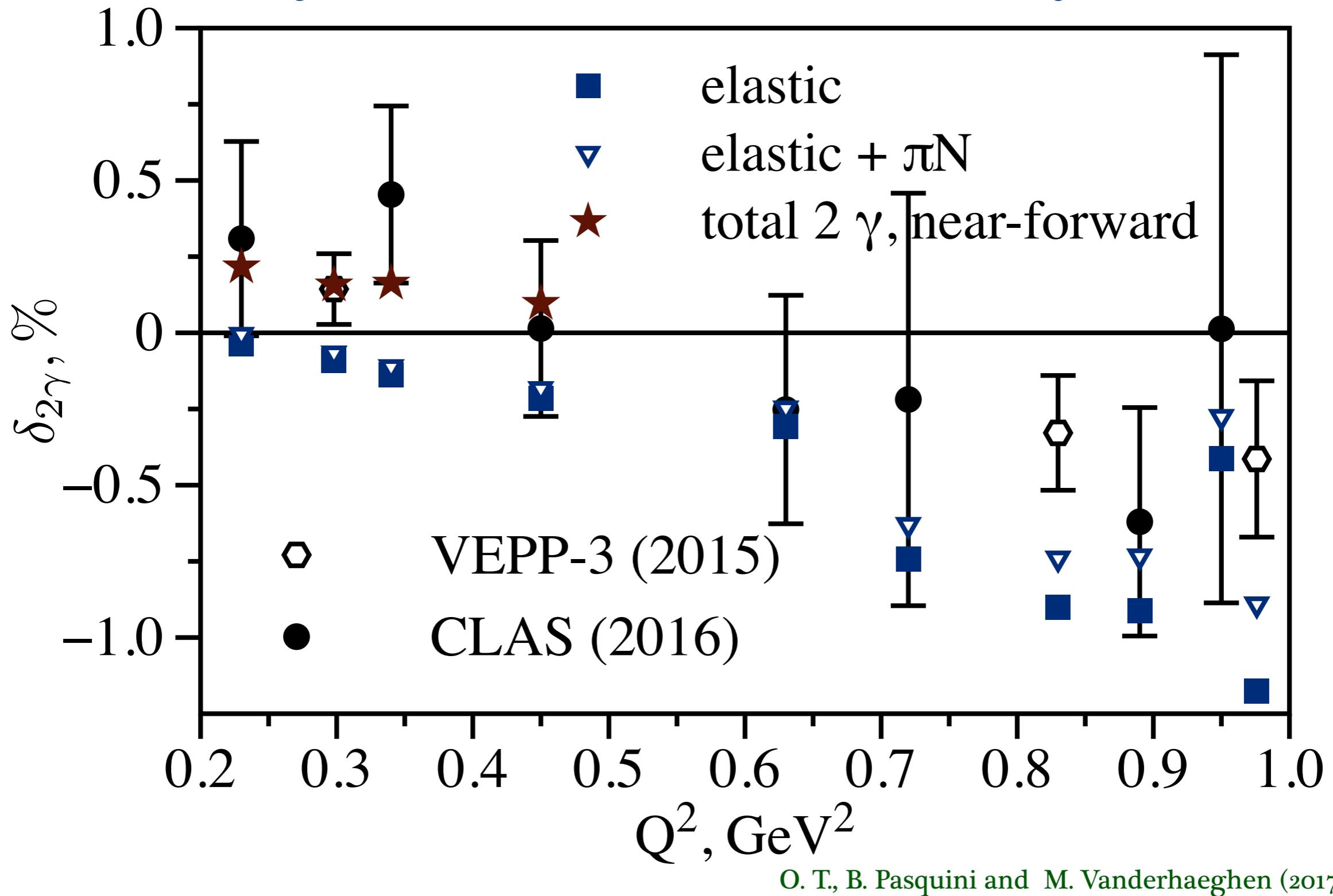


Comparison with data



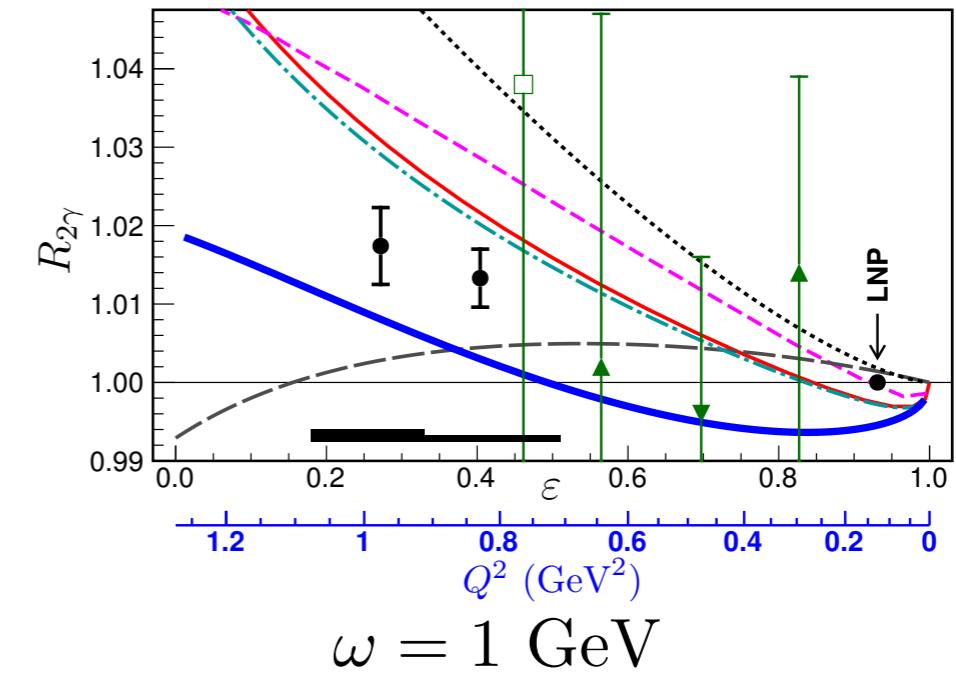
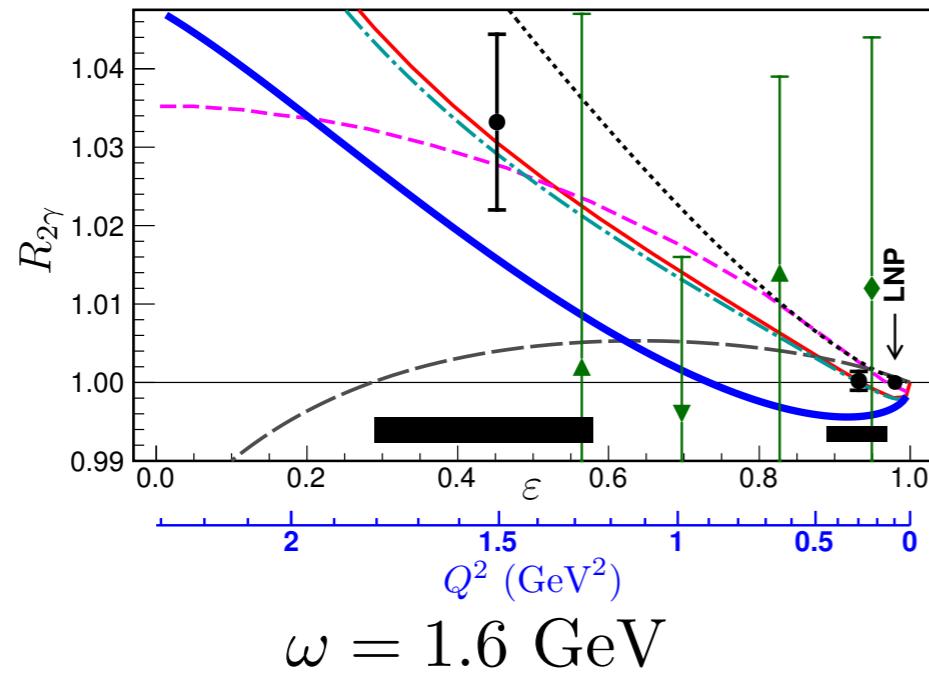
- near-forward 2γ agree with data
- multi-particle 2γ , e.g. $\pi\pi N$, is important

Comparison with data



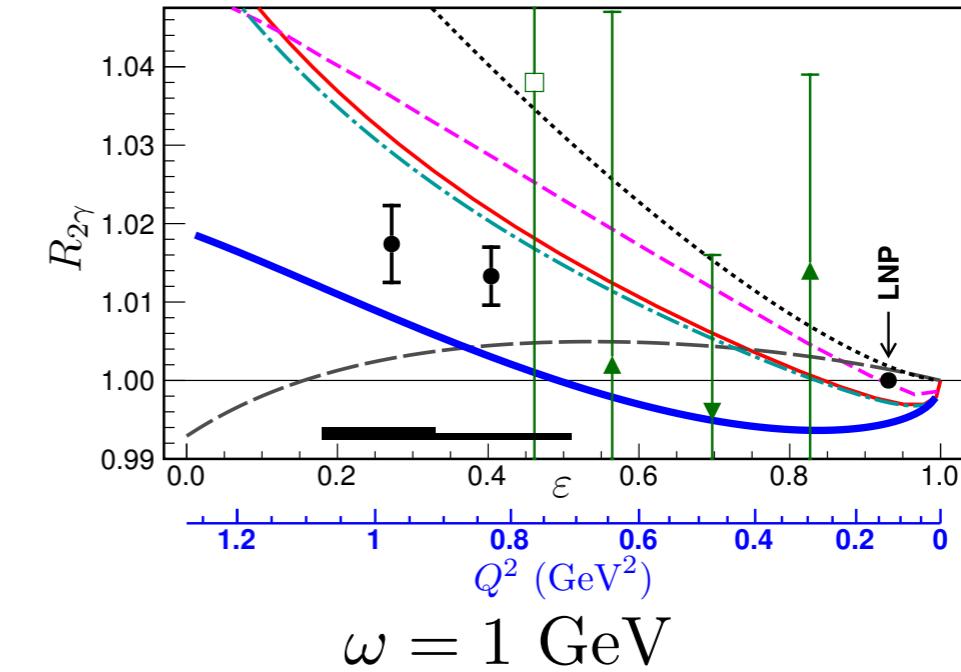
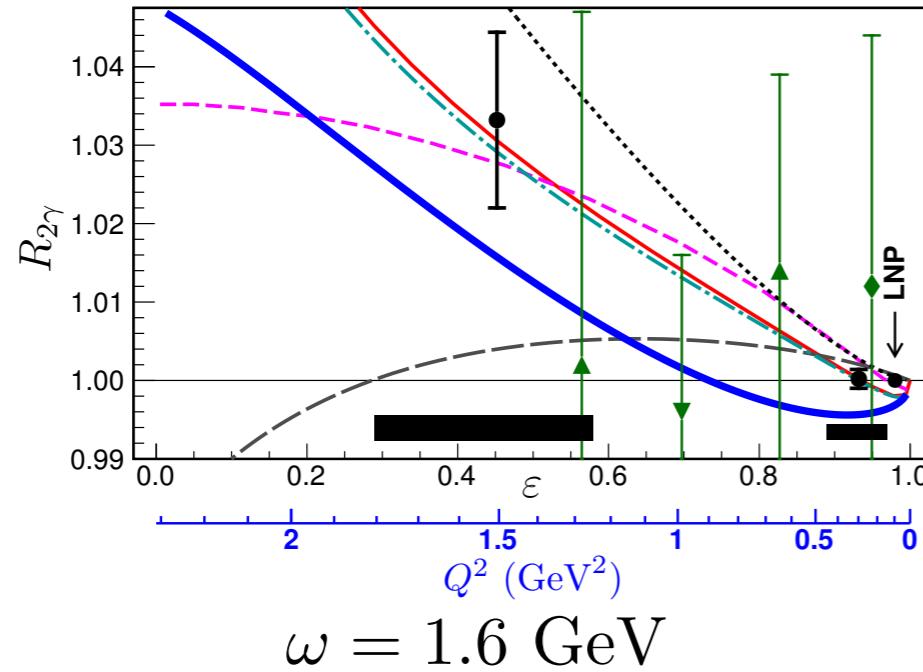
- dispersion relations agree with CLAS data

VEPP-3 and CLAS data

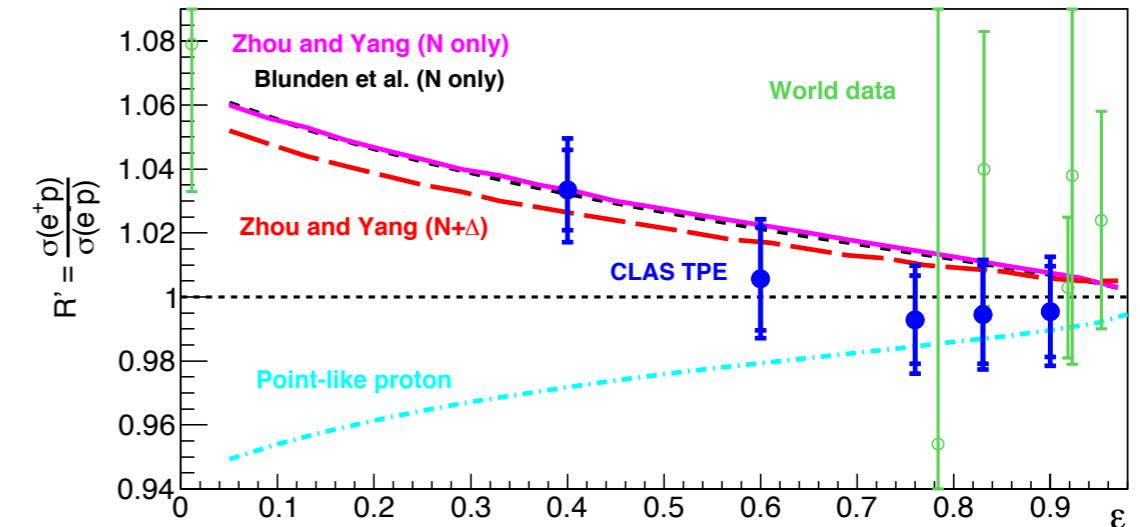
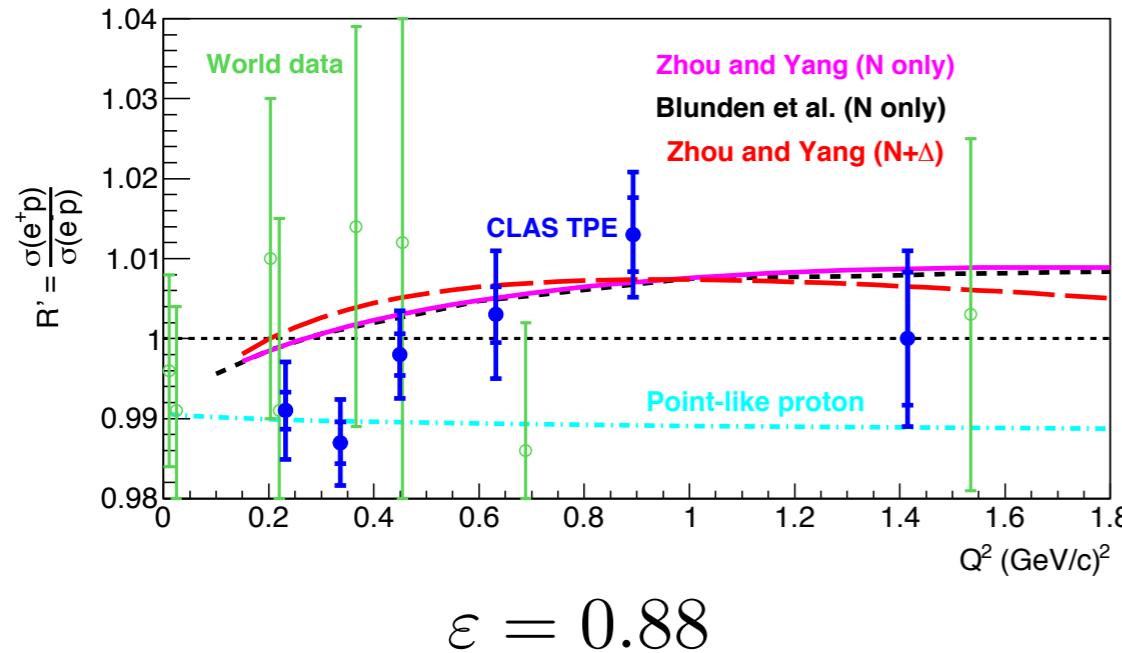


VEPP-3@Novosibirsk: I. A. Rachel et al. (2015)

VEPP-3 and CLAS data



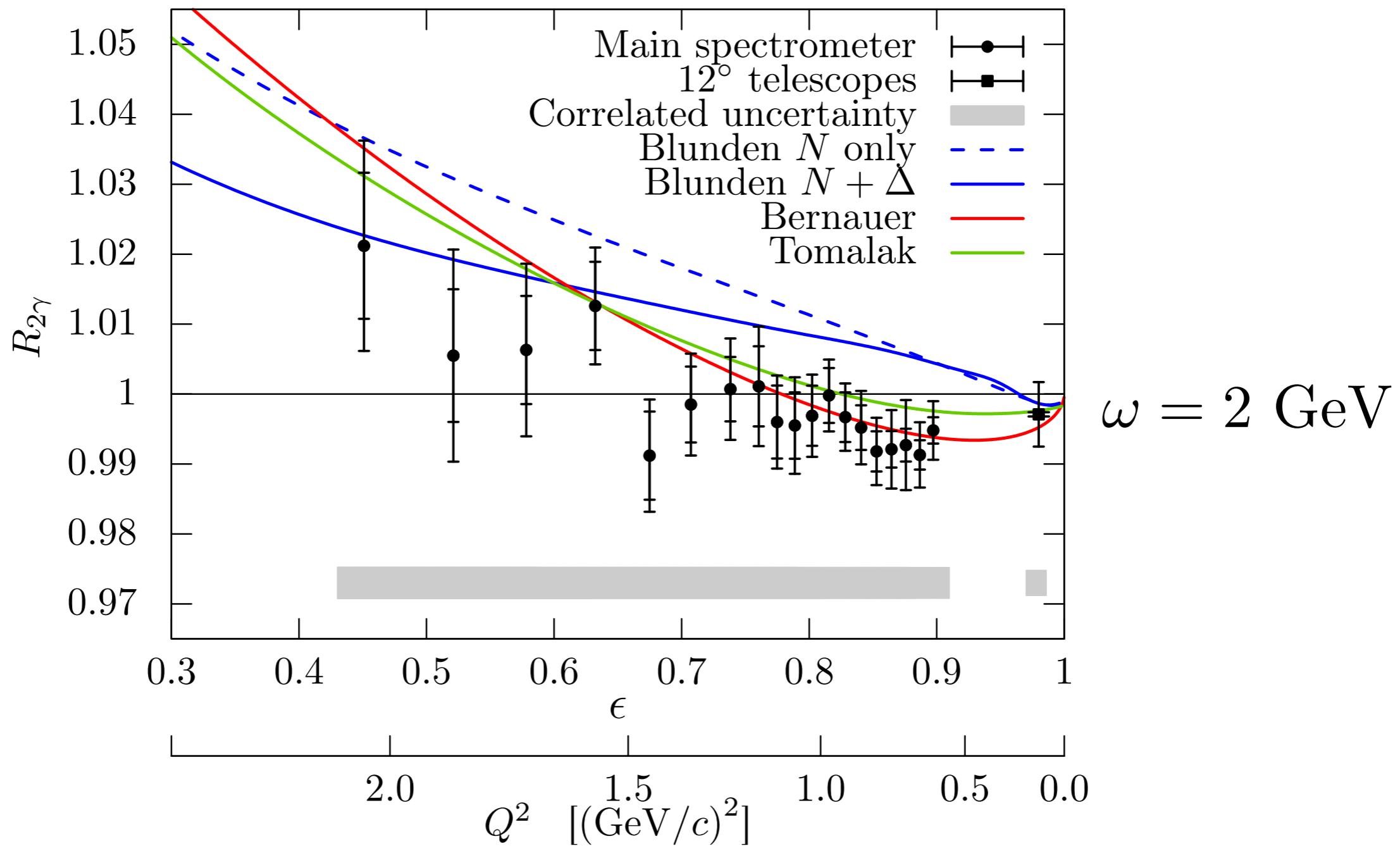
VEPP-3@Novosibirsk: I. A. Rachel et al. (2015)



CLAS@JLab: D. Adikaram et al. (2015)

- 2γ effect within $2-3\sigma$
- reasonable agreement theory vs. experiment

OLYMPUS data

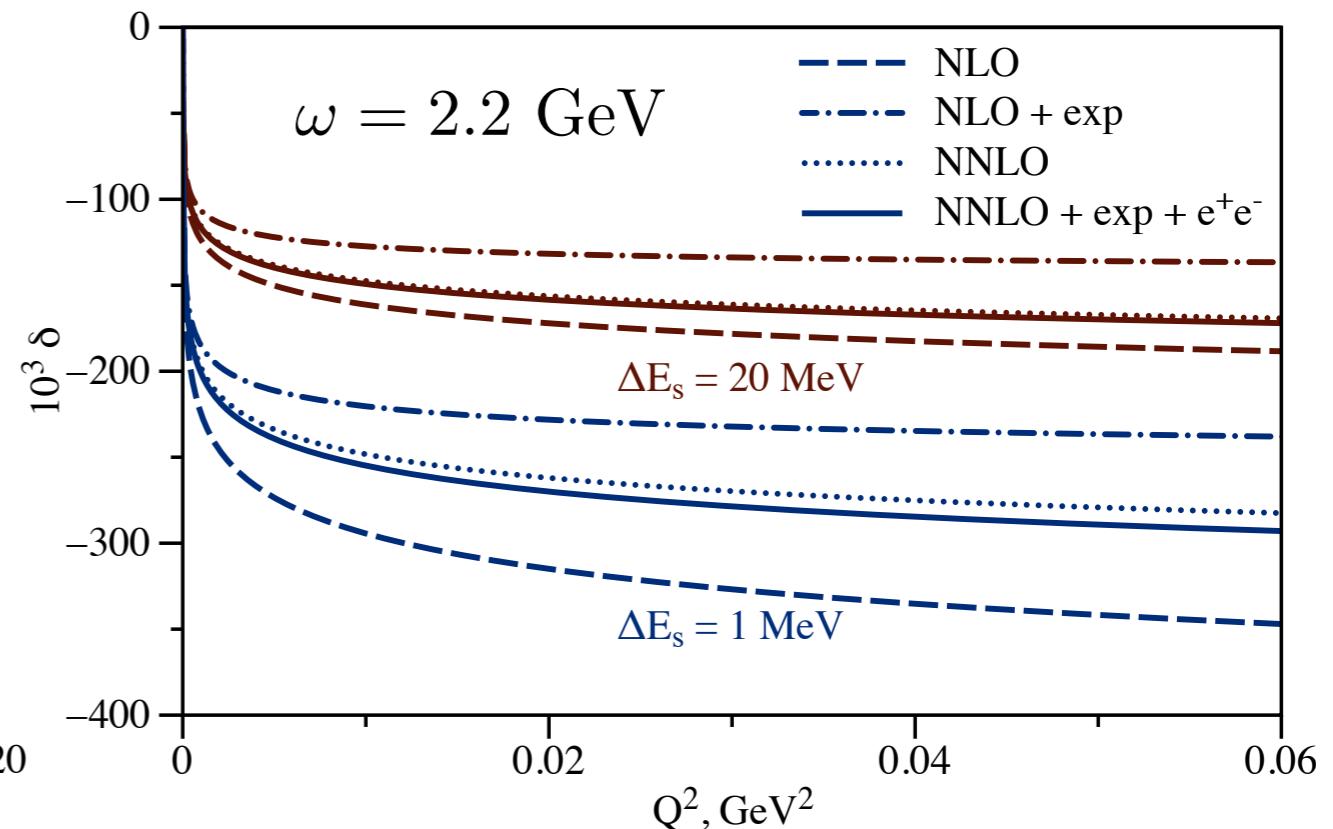
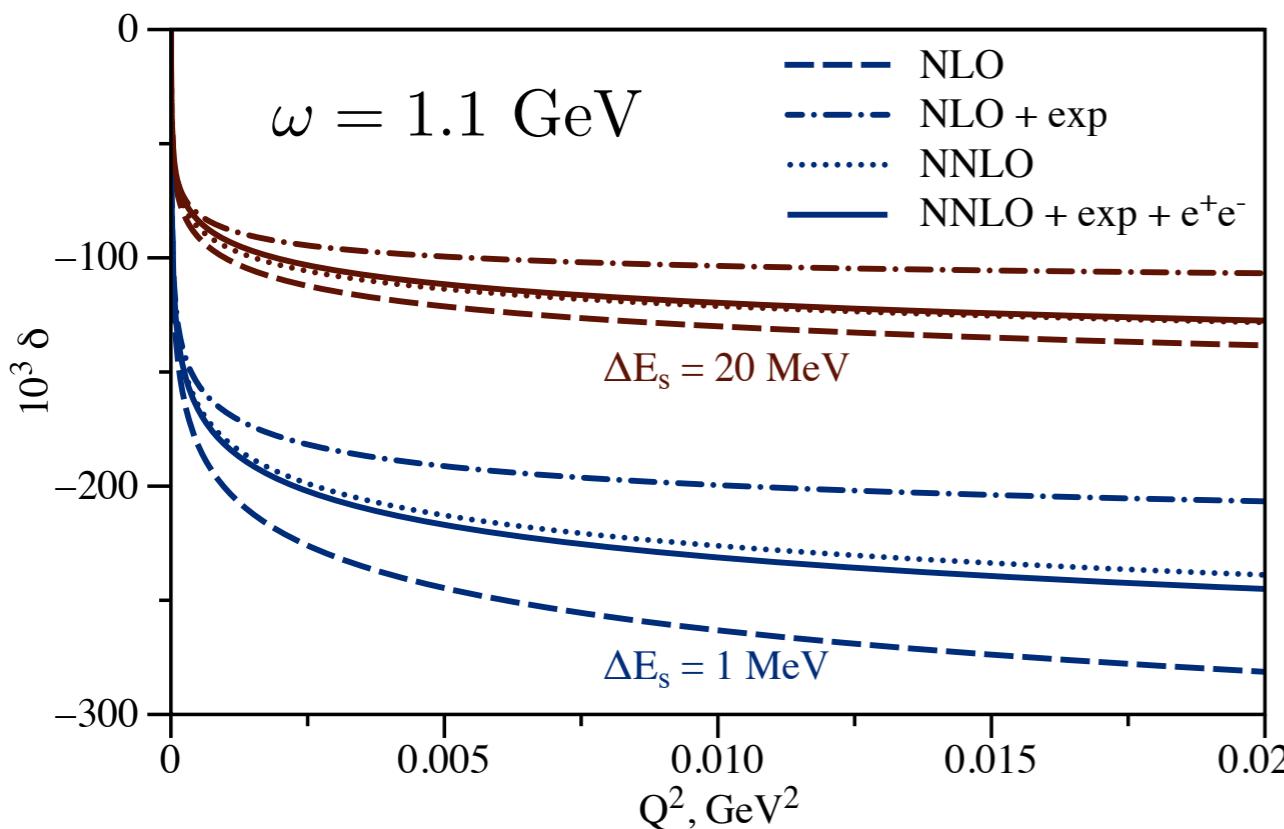


OLYMPUS@DESY: B. Henderson et al. (2017)

- in agreement with phenomenology
- radiative corrections are important

Estimate of higher orders

- cancellation of big logarithms: bremsstrahlung vs vertex correction
- exponentiation of soft logarithm from electron side + phase space
D. R. Yennie et al. (1961)
- two-loop QED vertex and 2γ bremsstrahlung
G. J. H. Burgers (1985), A. B. Arbuzov et al. (1998)

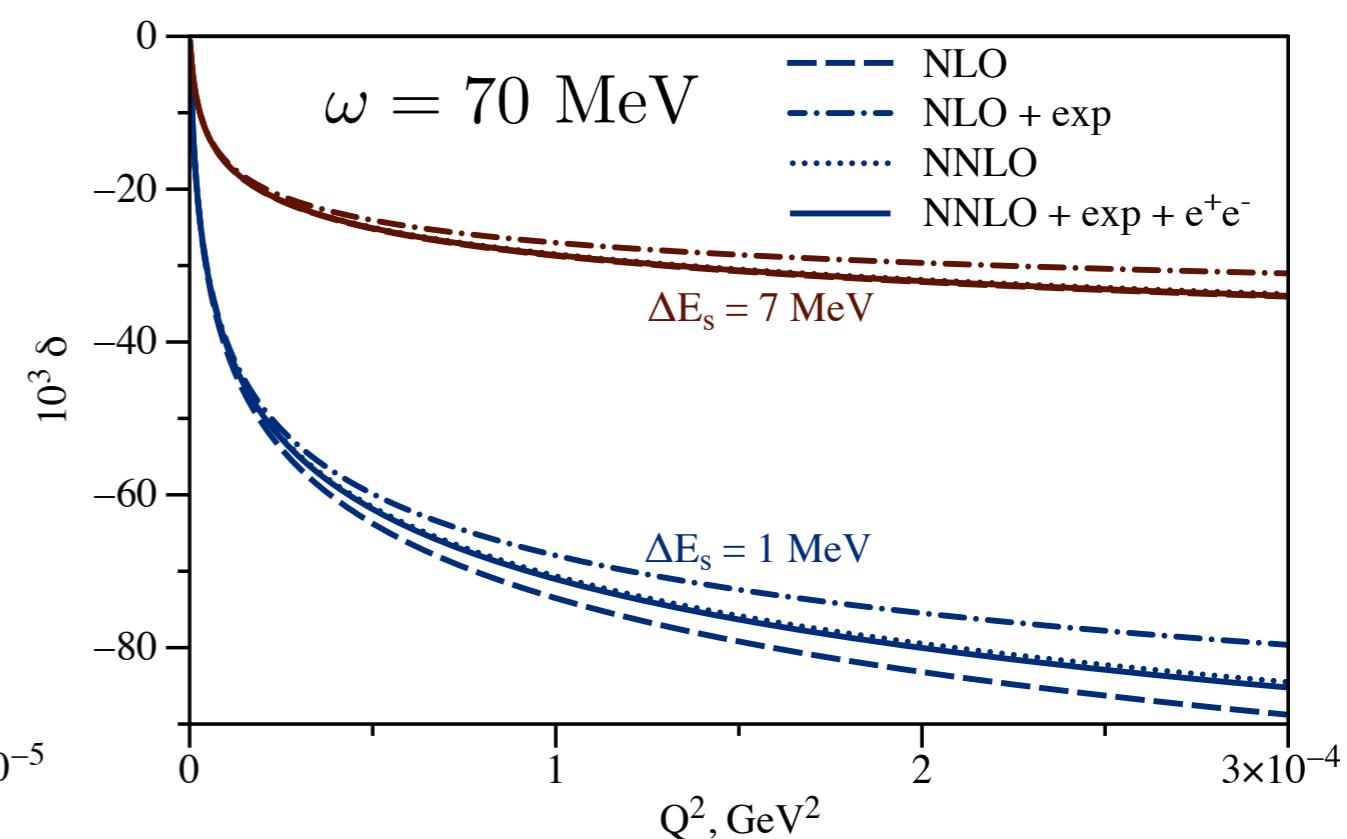
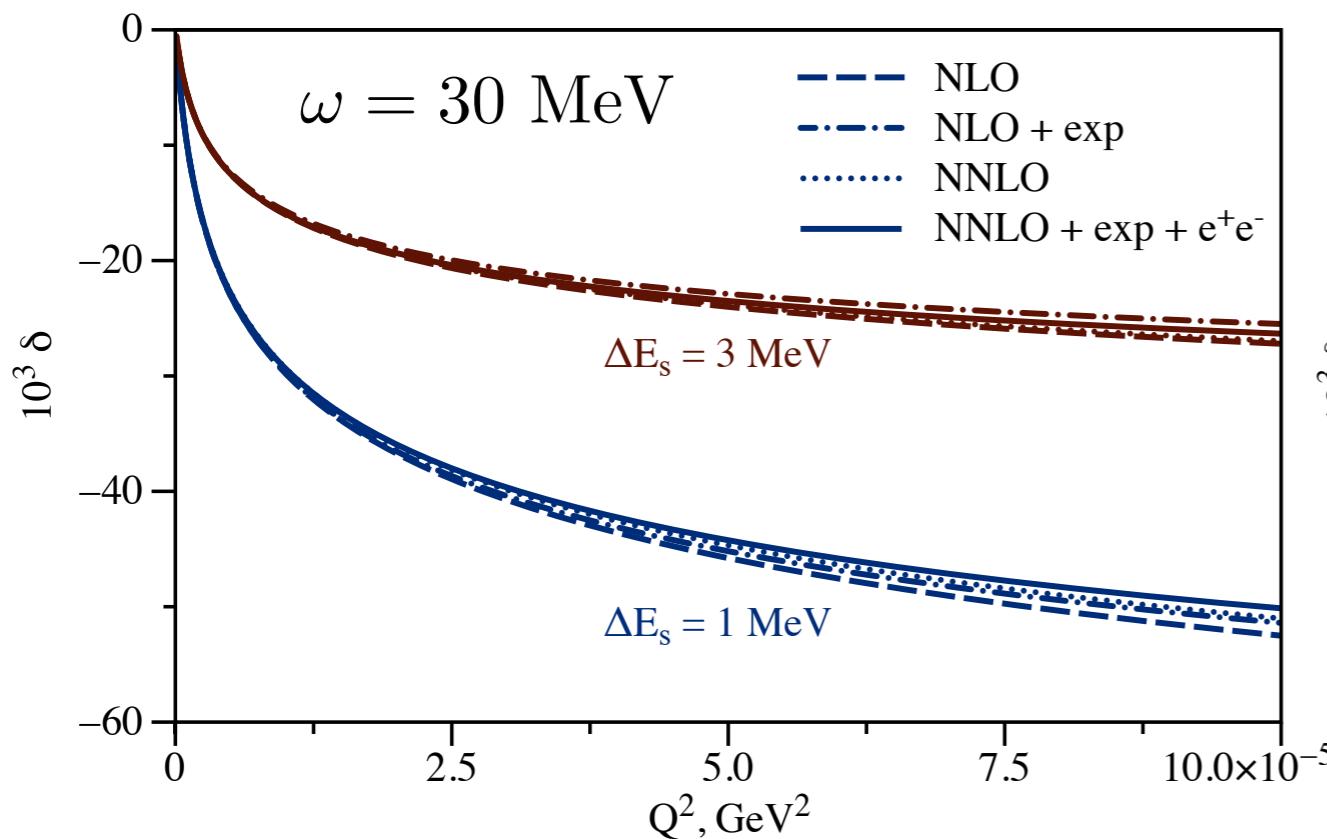


- large logarithm $\ln \frac{Q^2}{m^2}$ resummation: QED structure functions
F. A. Berends et al. (1988), A. B. Arbuzov (1998-2015) SCET
R. J. Hill (2017)
- account explicitly for next order: 5 % from NLO, 0.5 % from exp
E. A. Kuraev and V. S. Fadin (1985)

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SCET
R. Hill (2017)

- NLO + exp accuracy is sufficient for ProRad



Elastic muon-proton scattering and two-photon exchange



Elastic muon-proton scattering

- charge radius extractions:

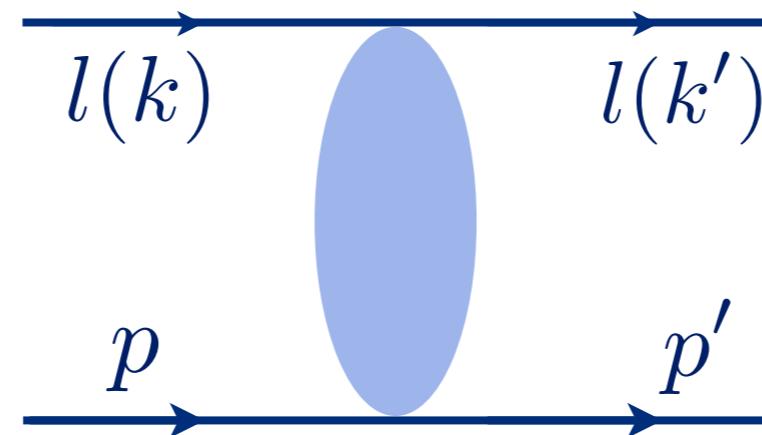
eH, eD spectroscopy	ep scattering
μ H, μ D spectroscopy	μ p scattering ???

- μ p elastic scattering is planned by MUSE@PSI(ongoing)
measure with both electron/muon charges
- three nominal beam energies: 115, 153, 210 MeV, $Q^2 < 0.1 \text{ GeV}^2$

- 2γ correction in MUSE ?

Elastic lepton-proton scattering and 2γ

$$K = \frac{k + k'}{2}$$



$$P = \frac{p + p'}{2}$$

- electron-proton scattering: 3 structure amplitudes

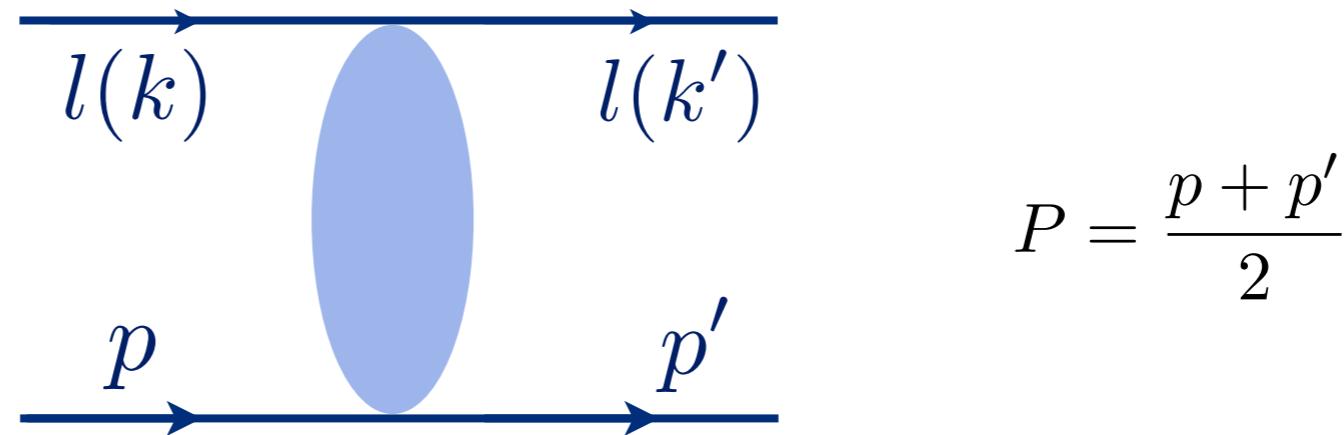
$$T^{\text{non-flip}} = \frac{e^2}{Q^2} \bar{l} \gamma_\mu l \cdot \bar{N} \left(\mathcal{G}_M(\nu, Q^2) \gamma^\mu - \mathcal{F}_2(\nu, Q^2) \frac{P^\mu}{M} + \mathcal{F}_3(\nu, Q^2) \frac{\hat{K} P^\mu}{M^2} \right) N$$

P.A.M. Guichon and M. Vanderhaeghen (2003)

- 2γ correction to cross section is given by amplitudes real parts

Elastic lepton-proton scattering and 2γ

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P.A.M. Guichon and M. Vanderhaeghen (2003)

- muon-proton scattering: add helicity-flip amplitudes

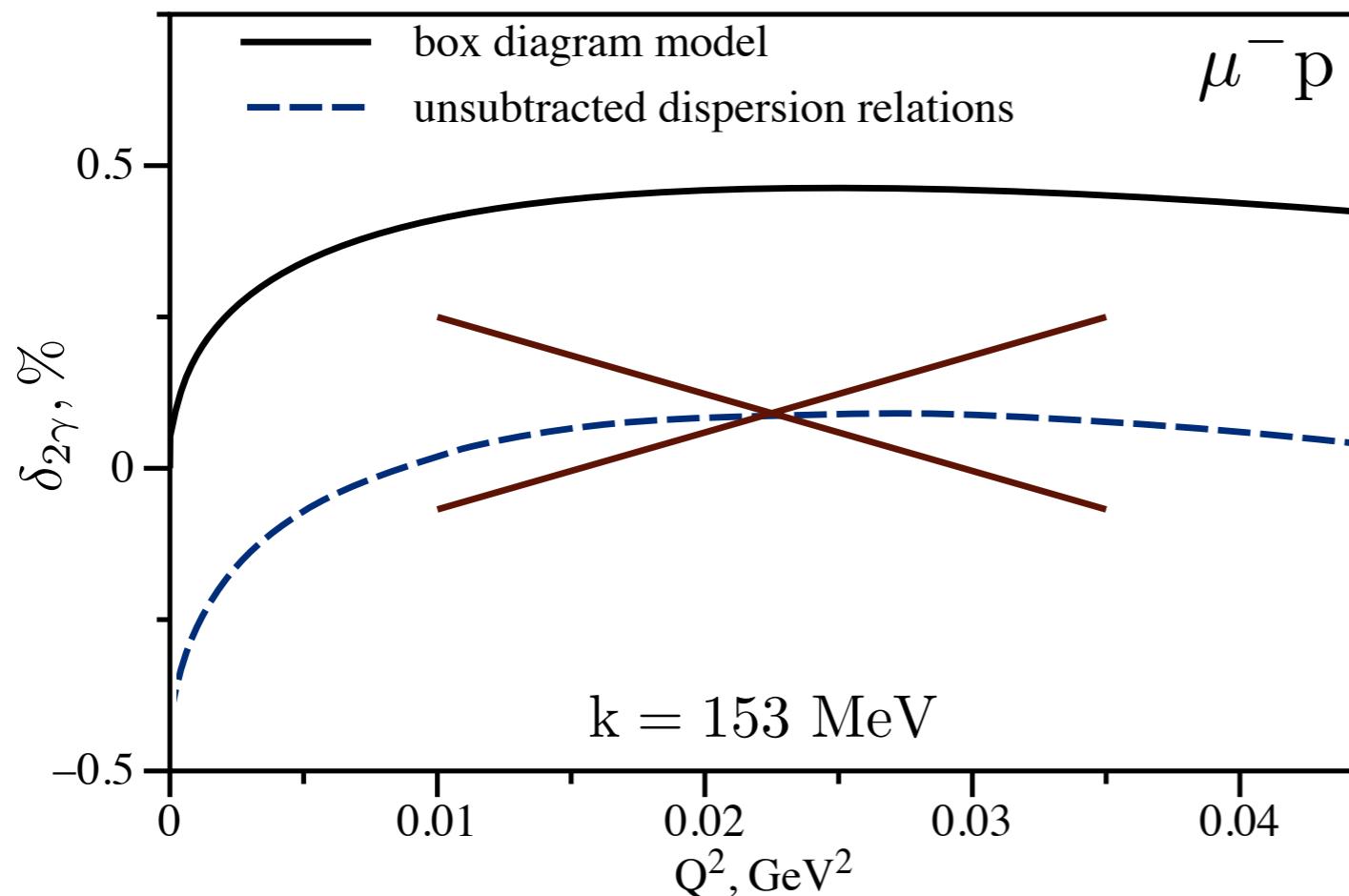
$$T^{\text{flip}} = \frac{e^2}{Q^2} \frac{m}{M} \bar{l} l \cdot \bar{N} \left(\mathcal{F}_4(\nu, Q^2) + \mathcal{F}_5(\nu, Q^2) \frac{\hat{K}}{M} \right) N + \frac{e^2}{Q^2} \frac{m}{M} \mathcal{F}_6(\nu, Q^2) \bar{l} \gamma_5 l \cdot \bar{N} \gamma_5 N$$

M. Gorchtein, P.A.M. Guichon and M. Vanderhaeghen (2004)

- 2γ correction to cross section is given by amplitudes real parts

Dispersion relation approach?

- proton state contribution to 2γ :



unsubtracted approach
violates low- Q^2 behavior

$$\delta_{2\gamma} \rightarrow 0$$

O. T. and M. Vanderhaeghen (2018)

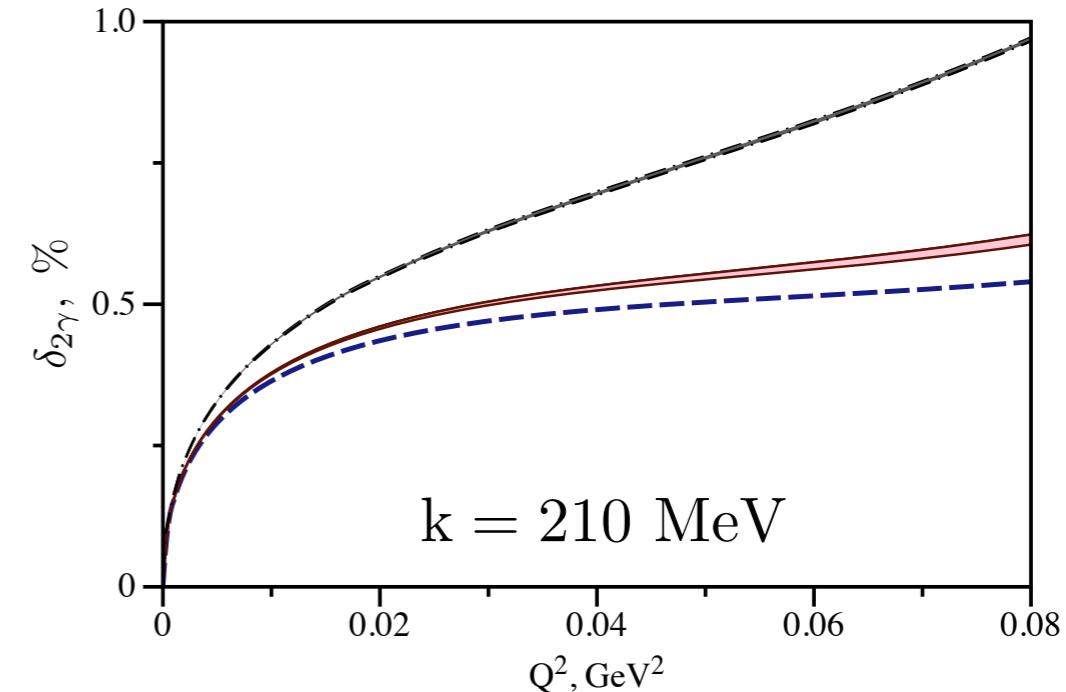
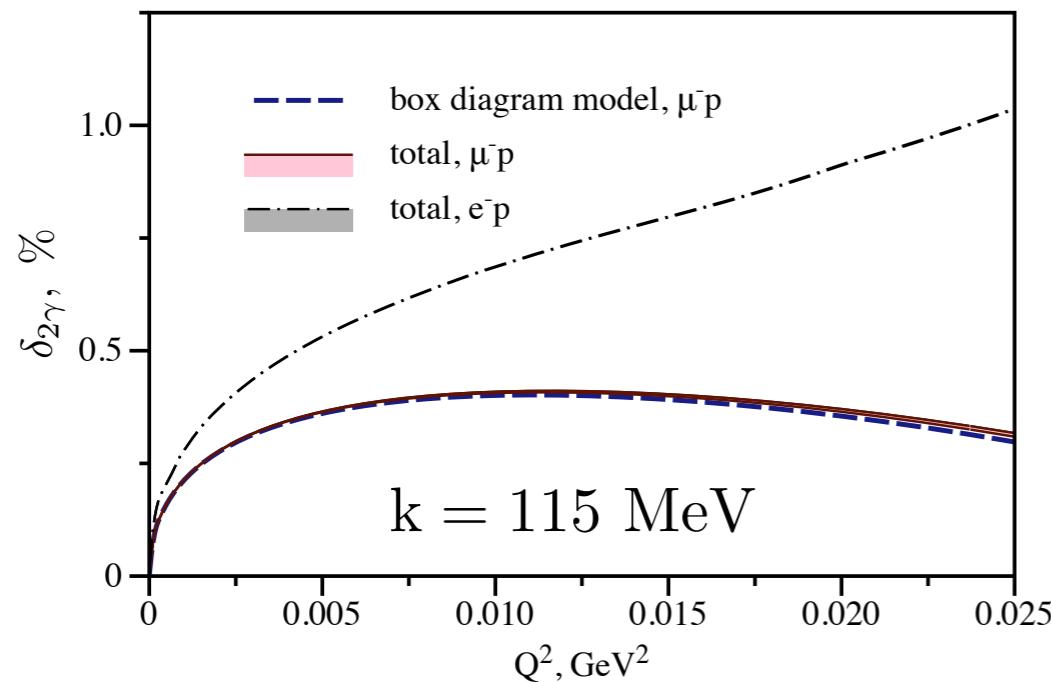
- problematic amplitude: \mathcal{F}_4

$\bar{l}l \cdot \bar{N}N$
structure

- dispersion relation approach requires a subtraction

MUSE@PSI (2018-19) estimates (μ^- -p)

- proton box diagram model + inelastic 2γ



O. T. and M. Vanderhaeghen (2014, 2016)

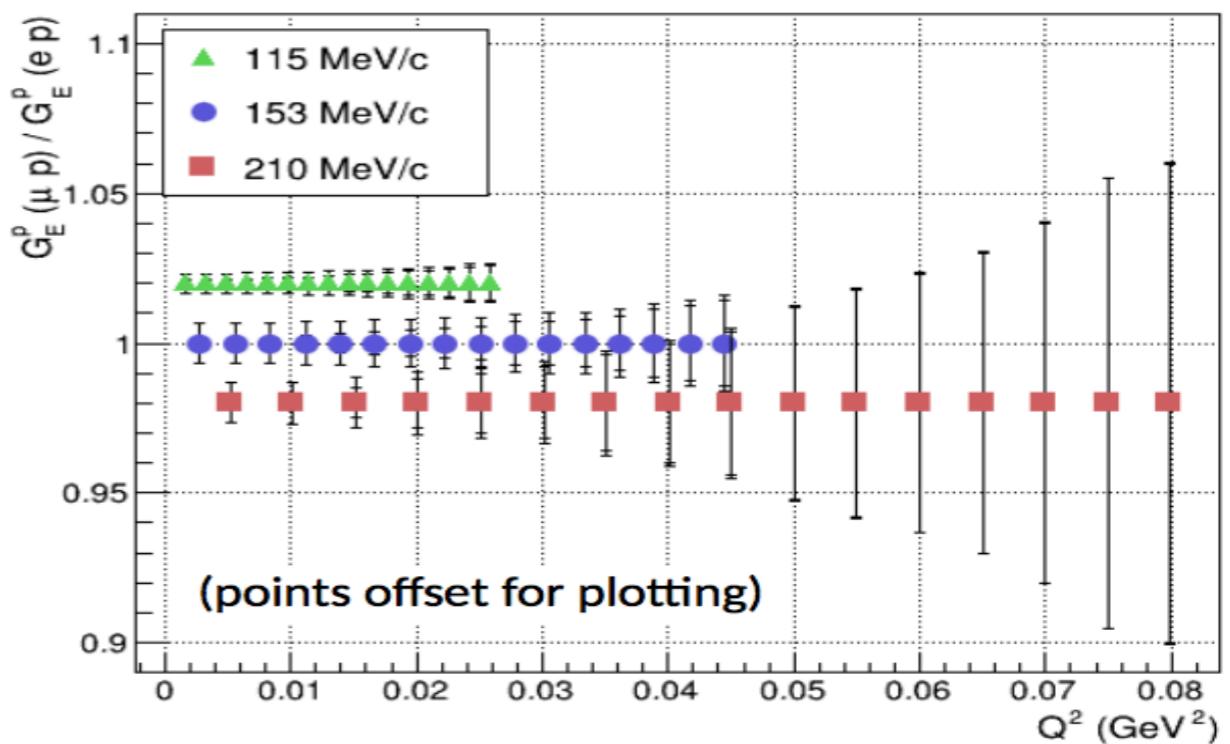
- expected muon over electron ratio

small inelastic 2γ



small 2γ uncertainty

- MUSE can test r_E in one charge channel



K. Mesick talk (PAVI 2014), MUSE TDR (2016)

COMPASS proton radius experiment

- elastic μp scattering at SPS with 100 GeV beam
- measure $G_E^2 + \tau G_M^2$ at forward angles
- data taking in 2022

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2 γ corrections?

- Feshbach correction (+ recoil)

$$\delta_{2\gamma} = \frac{\alpha\pi Q}{2\omega} \left(1 + \frac{m}{M}\right) \quad \rightarrow \quad \text{2-3 orders below MUSE}$$

- inelastic states: kinematically enhanced

- sub per mille level of 2γ in COMPASS kinematics

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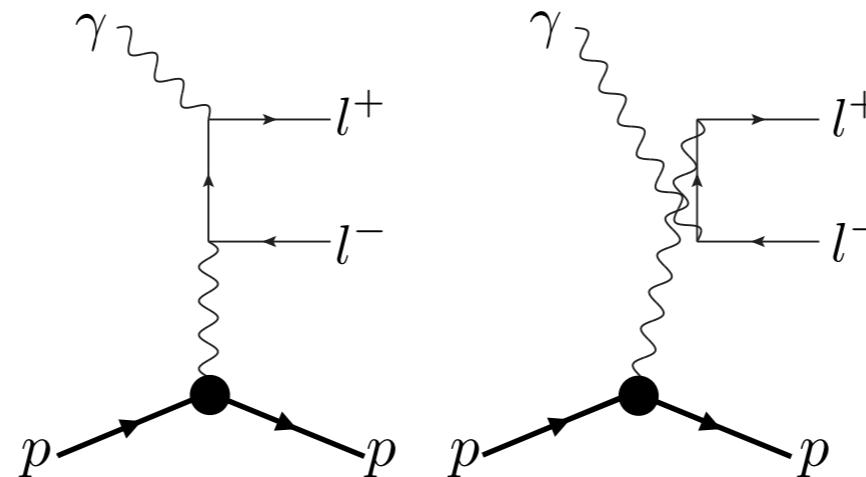
Lepton-pair production

- universality test by lepton-pair photoproduction:

$\gamma p \rightarrow l^+ l^- p$ @ MAMI

$\omega = 0.5 - 1.5$ GeV

$Q^2 = 0.0018 - 0.042$ GeV 2



$$\frac{\sigma(e^+e^-) + \sigma(\mu^+\mu^-)}{\sigma(e^+e^-)}$$

below and above
muon threshold

normalisation and proton structure errors are suppressed

Vl. Pauk and M. Vanderhaeghen (2015)

radiative corrections are presented in analytic form

M. Heller, O. T., M. Vanderhaeghen and Sh. Wu (2018-2019)

- one-loop QED is calculated
- 2γ vanishes averaging over lepton angles

Hyperfine splitting correction

- traditional decomposition of 2γ :

$$\Delta_{\text{HFS}} = \Delta_Z + \Delta^R + \Delta^{\text{pol}}$$

↓

Zemach term recoil correction polarizability
 G_E, G_M G_E, G_M F_2, g_1, g_2

- leading correction:

$$\Delta_Z = \frac{8\alpha m_r}{\pi} \int_0^\infty \frac{dQ}{Q^2} \left(\frac{G_M(Q^2) G_E(Q^2)}{\mu_P} - 1 \right)$$

- uncertainty budget:

> 100 ppm

< 10 ppm

100 ppm

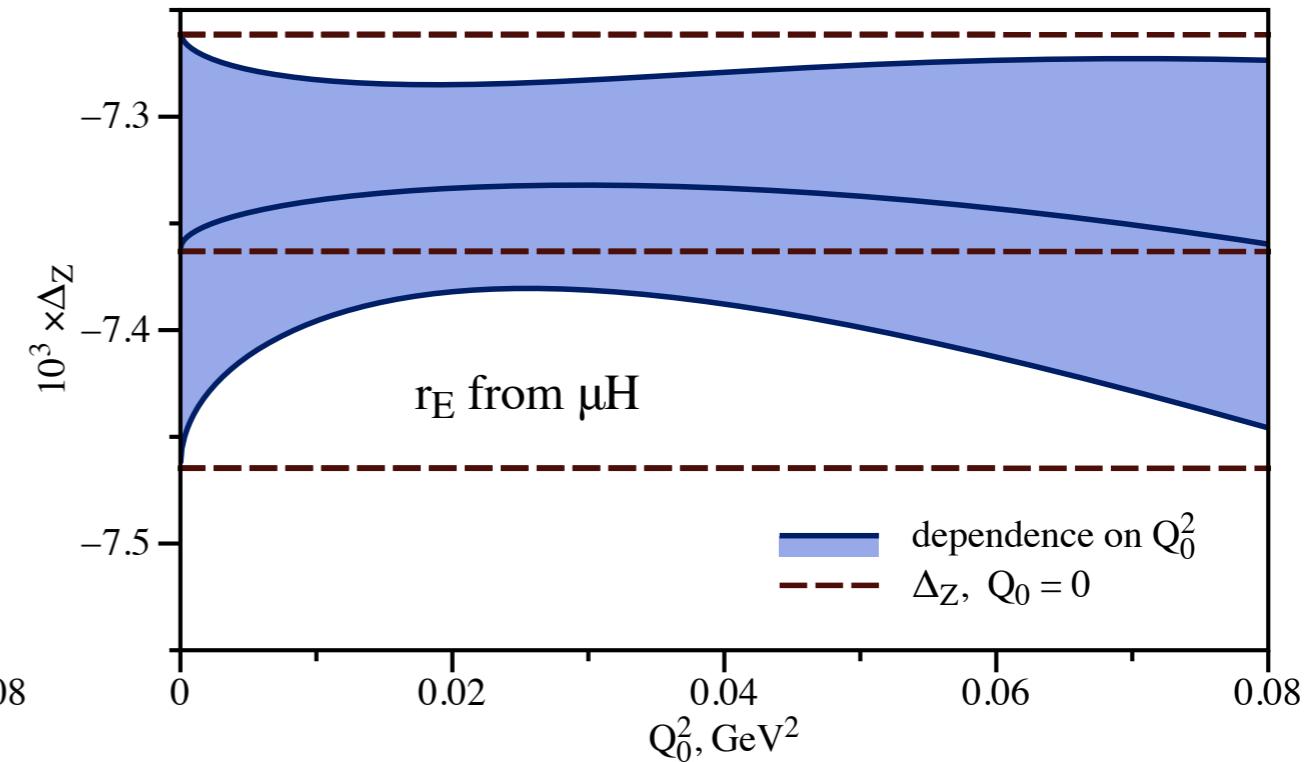
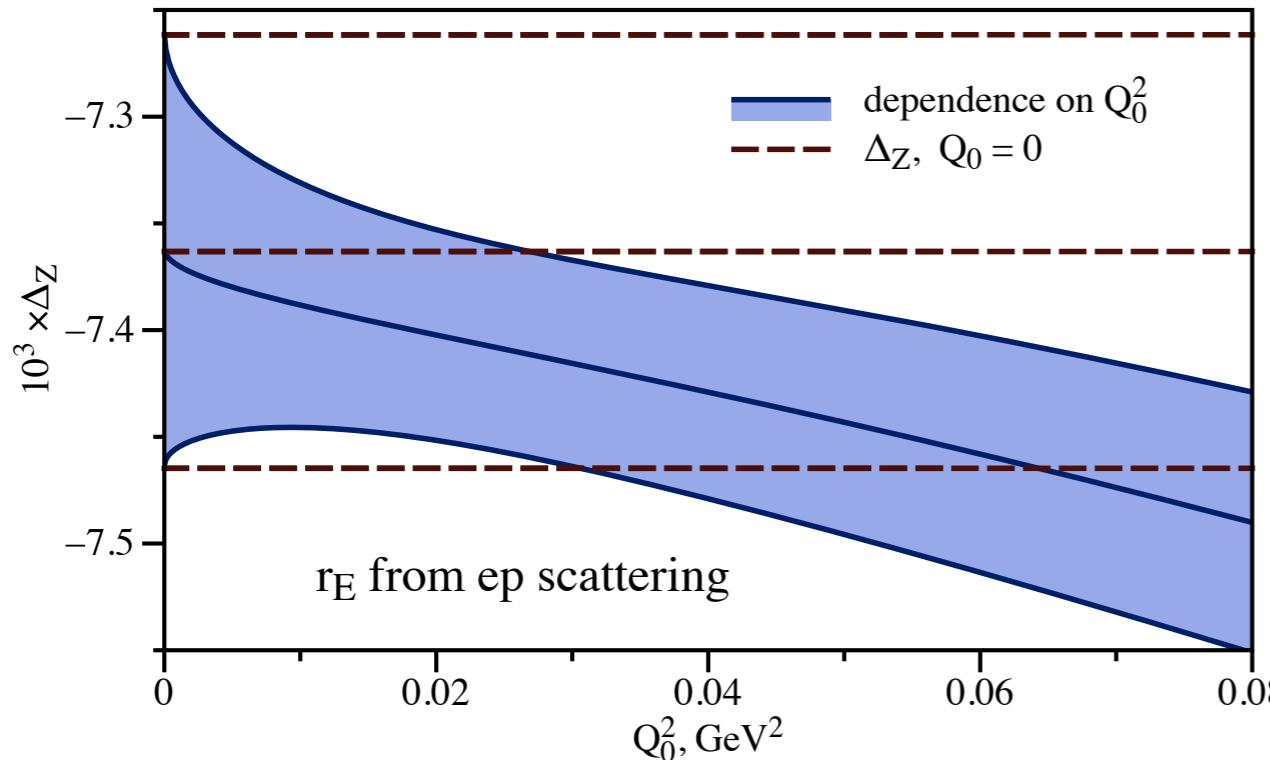
Zemach correction in μH

- Zemach correction expanding form factors:

$$\Delta_Z = \frac{8\alpha m_r}{\pi} \int_{Q_0}^{\infty} \frac{dQ}{Q^2} \left(\frac{G_M(Q^2) G_E(Q^2)}{\mu_P} - 1 \right) + \frac{4\alpha m_r Q_0}{3\pi} \left(-r_E^2 - r_M^2 + \frac{r_E^2 r_M^2}{18} Q_0^2 \right)$$

extraction of radii by Karshenboim (2014)

- dependence on splitting: consistency check



- 95 ppm change for μH and ep radii with $Q_0 = 0.2 \text{ GeV}$

O. T. (2017)

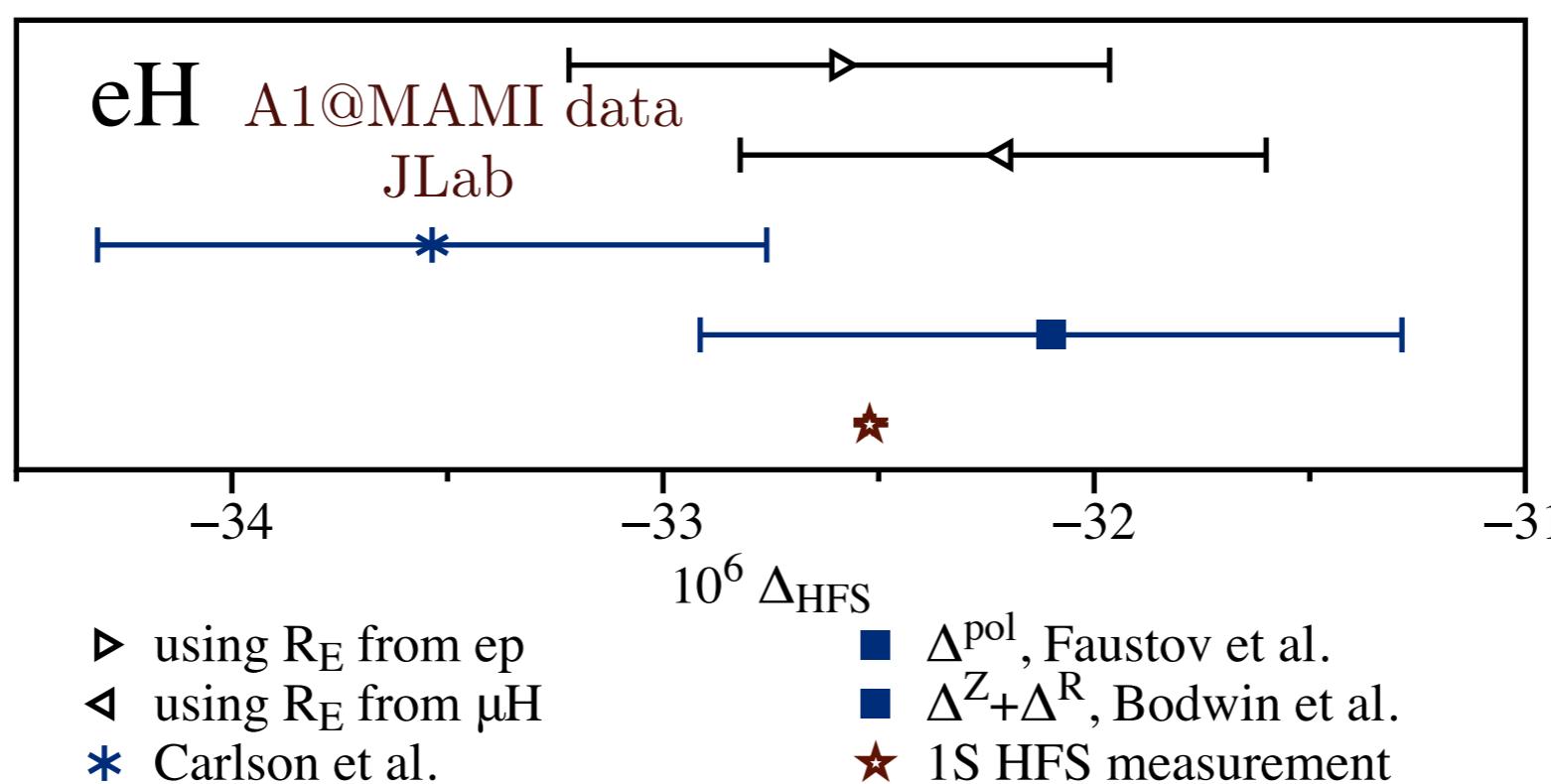
- 1.5-2 times more precise
- magnetic radius is equally important

2γ correction in eH 1S HFS

- measurements of 1S HFS in eH (21 cm line):

$$\nu_{\text{HFS}}(\text{H}) = 1420.4057517667(9) \text{ MHz} \quad \text{1970th}$$

- accuracy 10^{-12}
- precise extraction of 2γ



higher order corrections

M. I. Eides et al. (2008)
A. P. Martynenko et al.

error
 $\alpha \Delta_{\text{HFS}}$

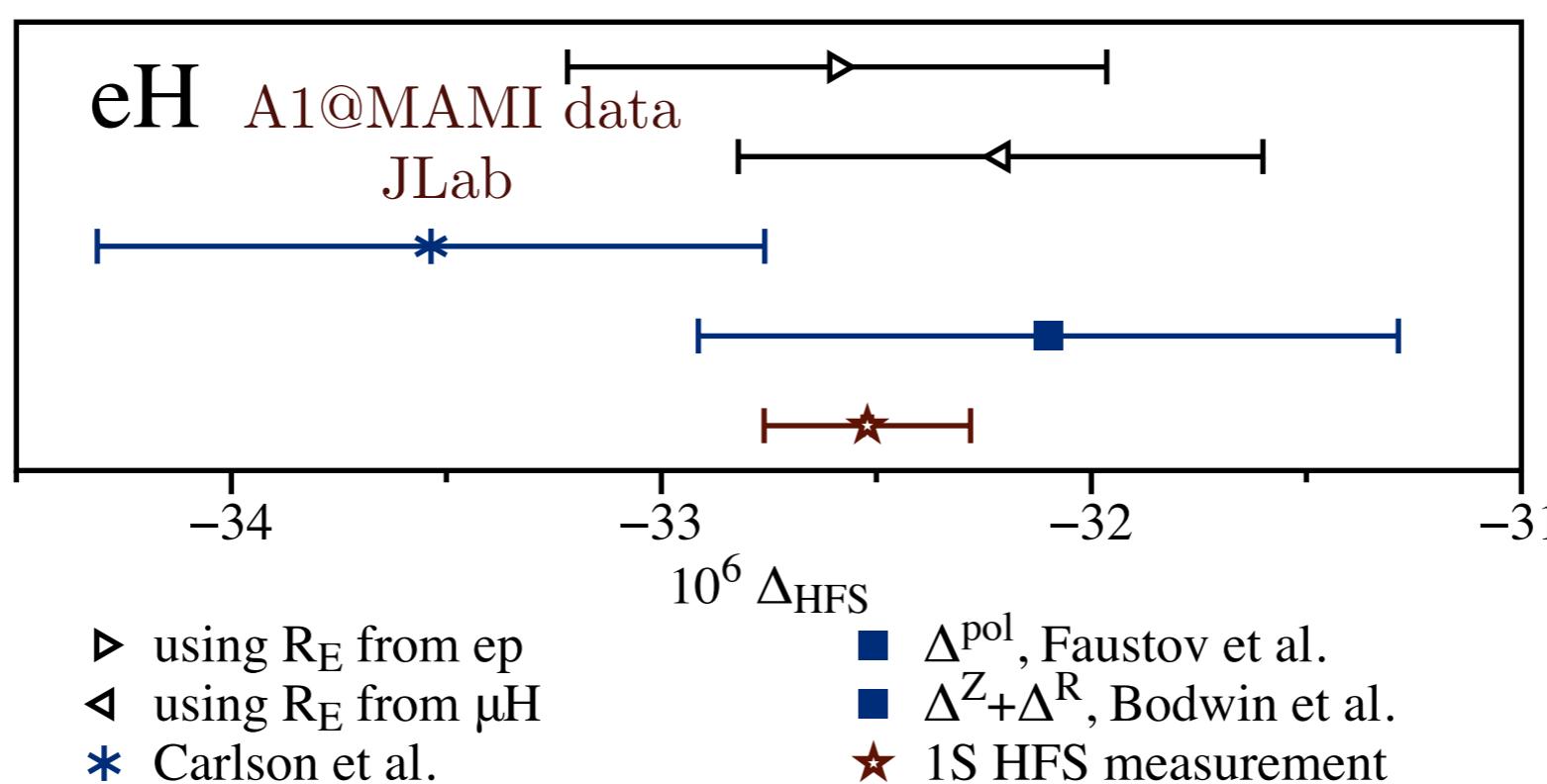
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Hyperfine splitting correction

- traditional decomposition of 2γ :

$$\delta E_{\text{HFS}}^{2\gamma} = \Delta_{\text{HFS}} E_F$$

$$\Delta_{\text{HFS}} = \Delta_Z + \Delta^R + \Delta^{\text{pol}}$$

Zemach term

$$G_E, G_M$$

recoil correction

$$G_E, G_M$$

polarizability

$$F_P, g_1, g_2$$

- leading correction:

$$\Delta_Z = \frac{8\alpha m_r}{\pi} \int_0^\infty \frac{dQ}{Q^2} \left(\frac{G_M(Q^2) G_E(Q^2)}{\mu_P} - 1 \right) \quad \Delta_{\text{pol}} \sim m_r$$

- uncertainty budget:

> 100 ppm

< 10 ppm

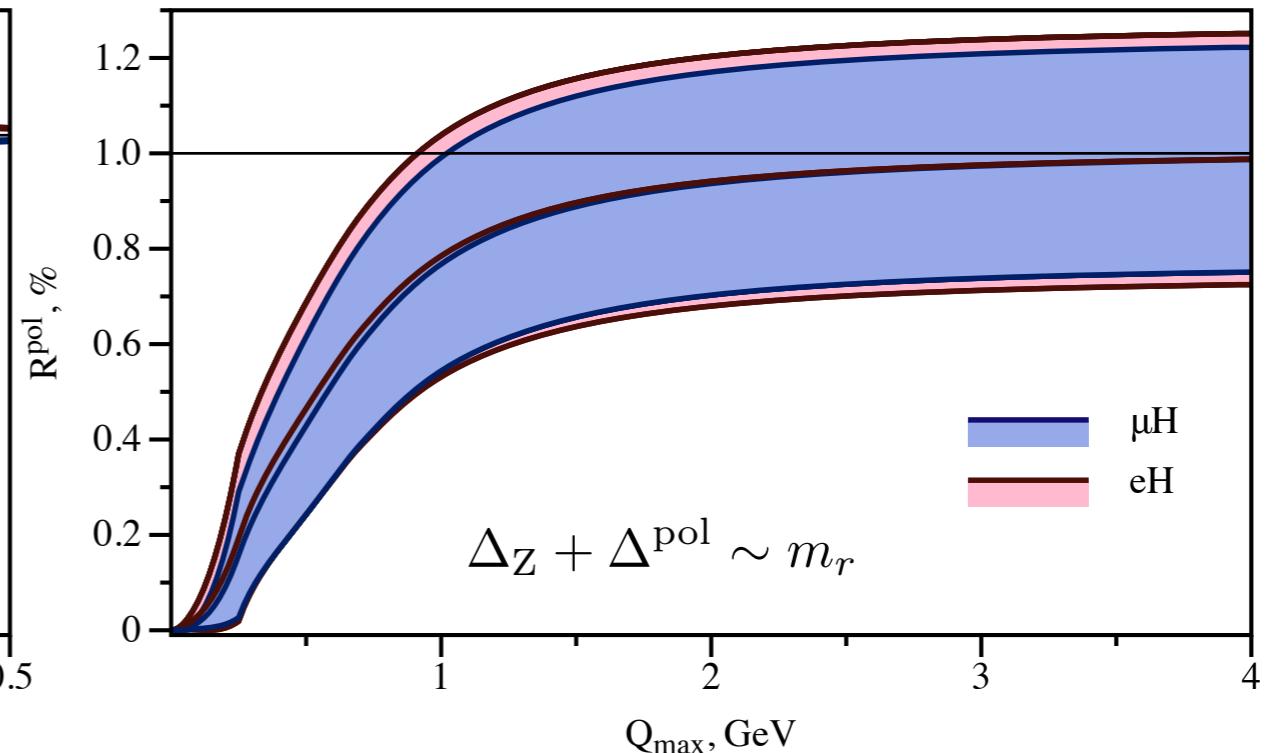
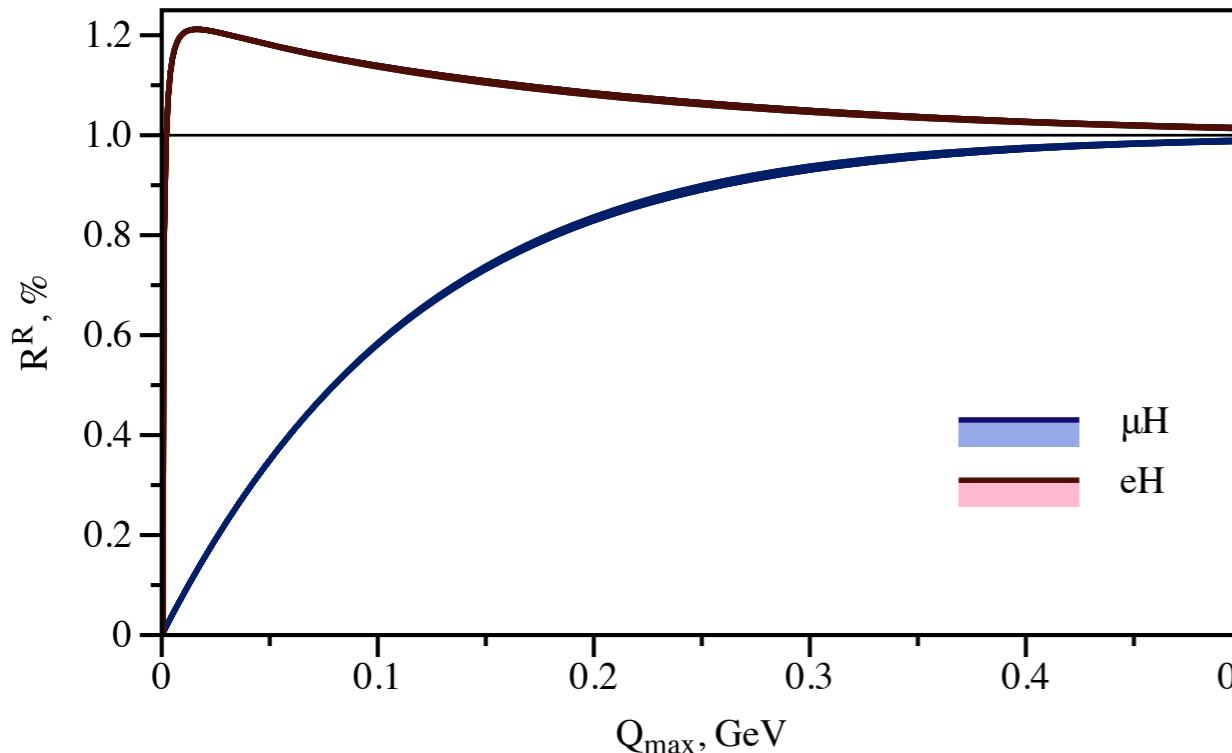
100 ppm

Connection between eH and μ H

- saturation of Q-integrals:

$$R^i = \frac{\Delta^i(Q_{\max})}{\Delta^i} = \int_0^{Q_{\max}} I^i(Q) dQ / \int_0^{\infty} I^i(Q) dQ$$

- Zemach correction: proportional to reduced mass



$$\Delta_{\text{HFS}}(\mu\text{H}) = \frac{m_r(m_\mu)}{m_r(m_e)} \Delta_{\text{HFS}}(\text{eH}) + \Delta_{\text{HFS}}^{\text{th}}(m_\mu) - \frac{m_r(m_\mu)}{m_r(m_e)} \Delta_{\text{HFS}}^{\text{th}}(m_e)$$

- Zemach correction vanishes and polarizability term is almost 0

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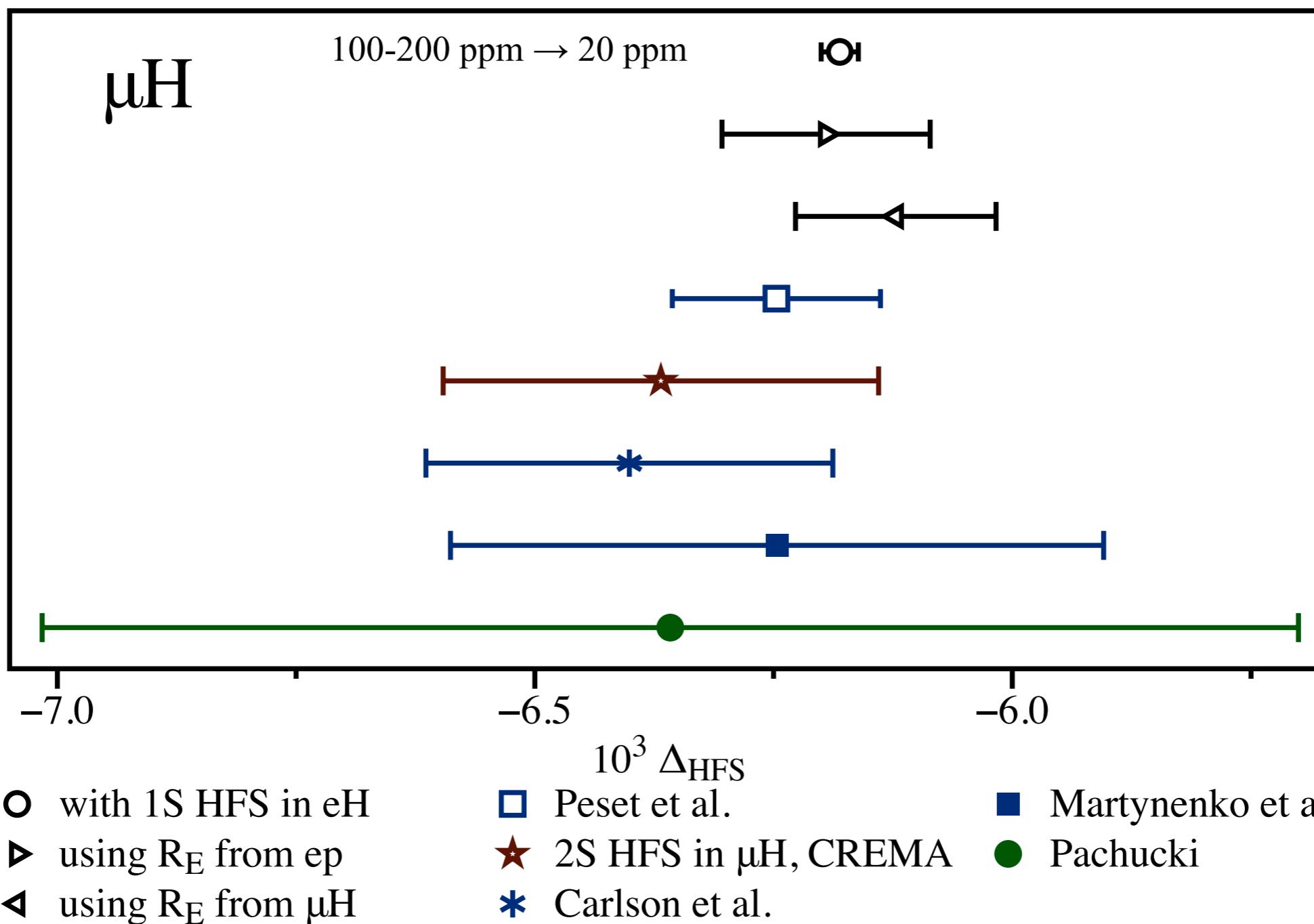
100 ppm

- μH splitting from eH :

$$\Delta_{\text{HFS}}(\mu H) = \frac{m_r(m_\mu)}{m_r(m_e)} \Delta_{\text{HFS}}(eH) + \Delta_{\text{HFS}}^{\text{th}}(m_\mu) - \frac{m_r(m_\mu)}{m_r(m_e)} \Delta_{\text{HFS}}^{\text{th}}(m_e)$$

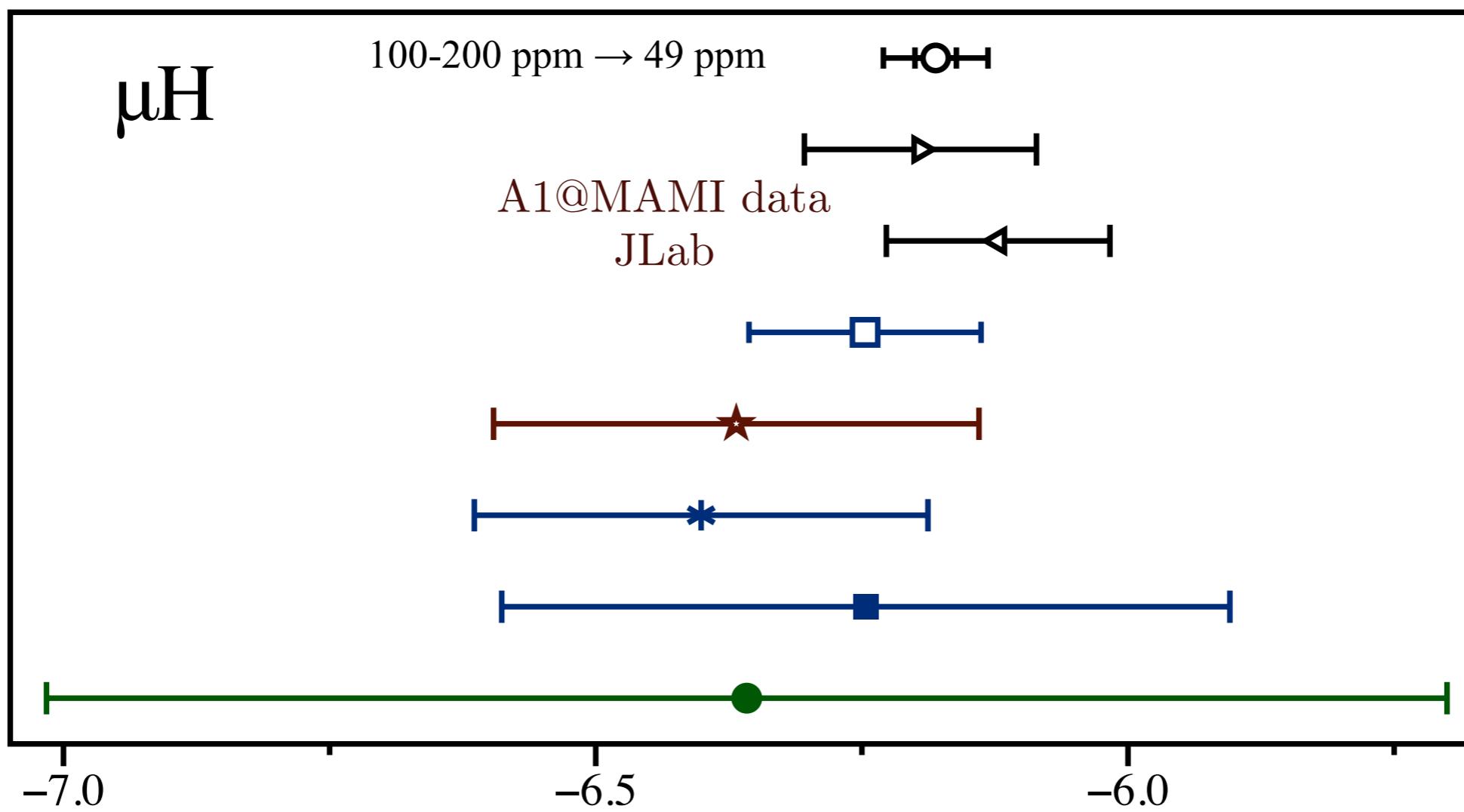
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2γ correction in μH from eH HFS



- error of 2γ is significantly reduced

2χ correction in μH from eH HFS



- with 1S HFS in eH
- ▷ using R_E from ep
- ◀ using R_E from μH

$10^3 \Delta_{\text{HFS}}$

Peset et al.

2S HFS in μH , CREMA

Carlson et al.

Martynenko et al.

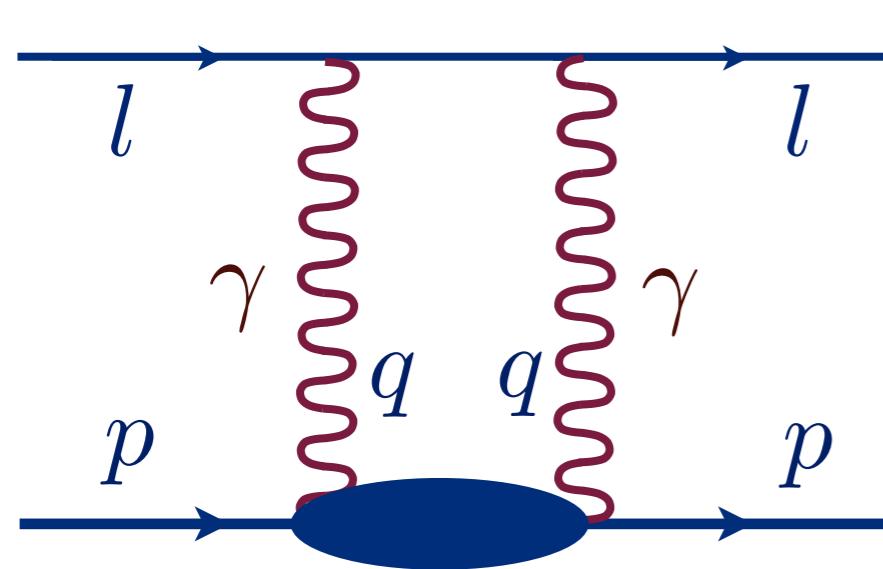
Pachucki

- precise S-level HFS prediction

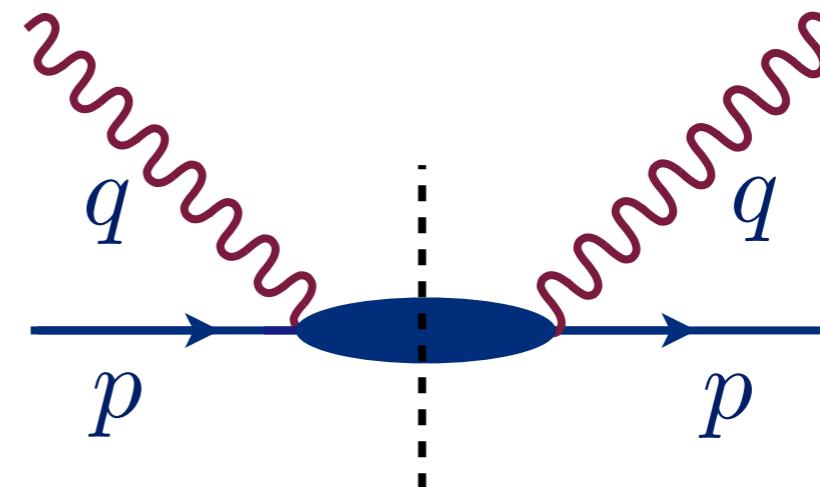
$$E_{1S}^{\text{HFS}} = 182.625 \pm 0.012 \text{ meV}$$

O. T. (2018, 2019)

Lamb shift 2γ correction. Forward VVCS



2γ blob - forward virtual Compton scattering



Optical theorem

$$\text{Im } T_1 \sim F_1$$

$$\text{Im } T_2 \sim F_2$$

$$\text{Im } S_1 \sim g_1$$

$$\text{Im } S_2 \sim g_2$$

Fixed- Q^2 dispersion relations

Disp. rel. for amplitude T_1 requires subtraction function

$$T_1^{\text{subt}}(0, Q^2) \equiv T_1(0, Q^2) - T_1^{\text{Born}}(0, Q^2)$$

Unsubtracted disp. rel. works for

$$T_2, S_1, S_2$$

Empirical estimate of subtraction function

Subtract Regge behavior + DR + resonance region and DIS data

G. Gasser, H. Leutwyler et al. (1974, 2015) M. Gorchtein et al. (2013) I. Caprini (2016)

expected low- Q^2 behavior

$$T_1^{\text{subtr}}(0, Q^2) = \beta(Q^2)Q^2$$

$$\beta(0) = \beta_M - \text{magnetic polarizability}$$

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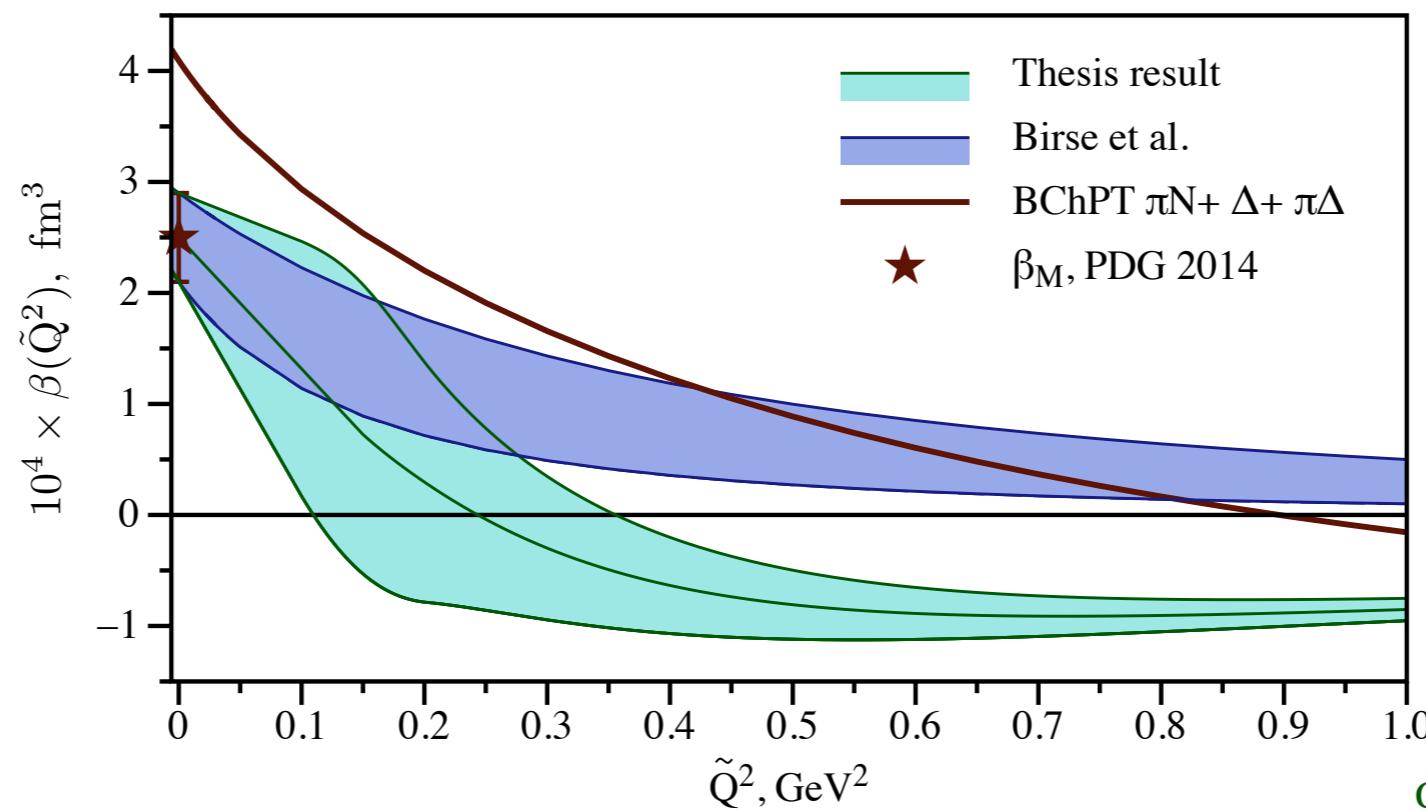
$$T_1^{\text{subtr}}(0, Q^2) = \beta(Q^2)Q^2$$

data-based result

vs.

$$\beta(0) = \beta_M - \text{magnetic polarizability}$$

theoretical predictions



Lamb shift correction

$$\Delta E_{2S}^{\text{subt}}(\mu H) \approx 2.3 \pm 1.3 \text{ } \mu\text{eV}$$

slightly smaller than Birse et al.

$$\Delta E_{2S}^{\text{subt}}(\mu H) \approx 4.2 \pm 1.0 \text{ } \mu\text{eV}$$